X Coord Y	Coord Label	Value	Type	e Historical
679924.815	3082872.349	J-47S	0	Manual
679994.969	3082983.51	J-48S	0	Manual
680057.658	3083072.075	J-49S	0	Manual
680077.354	3083115.533	J-50S	0	Manual
679827.115	3082729.746	J-51S	0	Manual
680141.873	3083080.88	J-52S	0	Manual
680170.56	3083064.674	J-53S	0	Manual

X Coord Y	Coord Label	Value	Type	e Historical
679530.943	3082575.448	G-36SD	0	Manual
679606.684	3082692.383	G-37SD	0	Manual
679671.317	3082565.925	G-46SD	0	Manual
679745.982	3082681.386	G-47SD	0	Manual
679521.779	3082672.022	J-54SD	0	Manual
680108.013	3083101.352	J-57SD	0	Manual
680413.831	3083297.013	J-58SD	0	Manual

Area: Area 1 X Coord Y Coord Label Value Type Historical 679638.466 G-21SD 0 Manual 3083412.561 679715.315 G-22SD 0 Manual 3083530.019 679789.831 3083644.936 G-23SD 0 Manual 679780.111 3083404.025 G-24SD 0 Manual 679854.225 3083519.822 G-25SD 0 Manual 679931.397 3083636.406 G-26SD 0 Manual G-27SD 0 679393.438 3082582.947 Manual G-28SD 0 Manual 679470.286 3082698.021 G-29SD 0 679543.314 3082816.106 Manual 679619.124 3082932.898 G-30SD 0 Manual G-31SD 0 679693.405 3083047.549 Manual 679768.226 3083162.727 G-32SD 0 Manual 3083280.235 G-33SD 0 Manual 679841.774 679917.766 3083397.416 G-34SD 0 Manual 679994.207 3083513.447 G-35SD 0 Manual G-36SD 0 Manual 679530.943 3082575.448 3082692.383 G-37SD 0 Manual 679606.684 679681.465 3082807.799 G-38SD 0 Manual Manual 679756.109 3082922.939 G-39SD 0 679832.158 3083041.871 G-40SD 0 Manual 679906.421 3083156.754 G-41SD 0 Manual G-42SD 0 Manual 679982.277 3083273.065 G-43SD 0 680056.981 3083387.33 Manual 680132.65 3083505.456 G-44SD 0 Manual 679597.188 3082450.923 G-45SD 0 Manual G-46SD 0 Manual 679671.317 3082565.925 G-47SD 0 679745.982 3082681.386 Manual G-48SD 0 Manual 679822.074 3082799.575 679896.612 3082915.666 G-49SD 0 Manual 679971.215 3083030.792 G-50SD 0 Manual G-51SD 0 Manual 680046.783 3083146.999 680123.432 3083263.001 G-52SD 0 Manual G-53SD 0 Manual 680198.058 3083379.815 680185.412 3083138.847 G-54SD 0 Manual 680260.421 3083254.807 G-55SD 0 Manual 680335.97 3083372.02 G-56SD 0 Manual Manual 680022.008 3083237.472 J-44SD 680047.095 3083215.942 J-45SD 0 Manual 679887.433 3082812.936 J-46SD Manual 0 679521.779 3082672.022 J-54SD 0 Manual 679590.092 3082773.184 J-55SD 0 Manual Manual 679763.399 3083050.055 J-56SD 0 680108.013 3083101.352 J-57SD 0 Manual 680413.831 3083297.013 J-58SD Manual

X Coord Y	Coord Label	Value	Type	e Historical
679924.815	3082872.349	J-47S	0	Manual
679994.969	3082983.51	J-48S	0	Manual
680057.658	3083072.075	J-49S	0	Manual
680077.354	3083115.533	J-50S	0	Manual
679827.115	3082729.746	J-51S	0	Manual
680141.873	3083080.88	J-52S	0	Manual
680170.56	3083064.674	J-53S	0	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

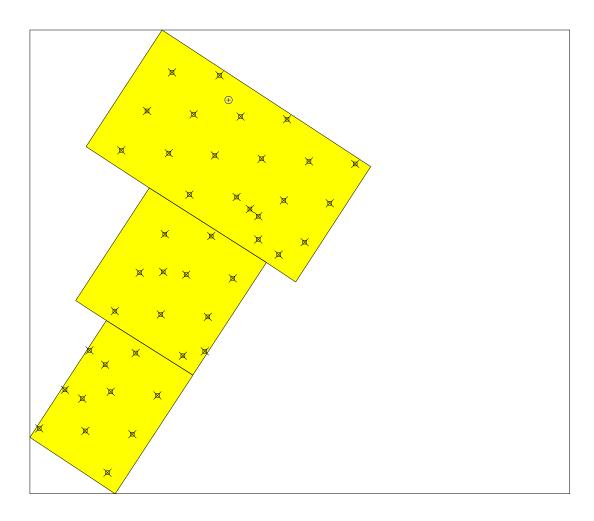
SUMMARY OF SAMPLING DESIGN						
Primary Objective of Design	Compare a site mean to a fixed threshold					
Type of Sampling Design	Parametric					
Sample Placement (Location) in the Field	Simple random sampling					
Working (Null) Hypothesis	The mean value at the site exceeds the threshold					
Formula for calculating number of sampling locations	Student's t-test					
Calculated total number of samples	15					
Number of samples on map <sup>a</sup>	44					
Number of selected sample areas b	3					
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>					
Total cost of sampling <sup>d</sup>	\$8,500.00					

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1								
X Coord	Y Coord	Label	Value	Туре	Historical			
679638.4660	3083412.5610	G-21SD	971	Manual	Т			
679715.3150	3083530.0190	G-22SD	34700	Manual	Т			
679789.8310	3083644.9360	G-23SD	35900	Manual	Т			
679780.1110	3083404.0250	G-24SD	2600	Manual	Т			
679854.2250	3083519.8220	G-25SD	5490	Manual	Т			
679931.3970	3083636.4060	G-26SD	6525	Manual	Т			
679841.7740	3083280.2350	G-33SD	3810	Manual	Т			
679917.7660	3083397.4160	G-34SD	4560	Manual	Т			
679994.2070	3083513.4470	G-35SD	2220	Manual	Т			
679982.2770	3083273.0650	G-42SD	1080	Manual	Т			
680056.9810	3083387.3300	G-43SD	13400	Manual	Т			
680132.6500	3083505.4560	G-44SD	1790	Manual	Т			
680046.7830	3083146.9990	G-51SD	2060	Manual	Т			
680123.4320	3083263.0010	G-52SD	1900	Manual	Т			
680198.0580	3083379.8150	G-53SD	5000	Manual	Т			
680185.4120	3083138.8470	G-54SD	2870	Manual	Т			

680260.4210	3083254.8070	G-55SD	12300	Manual	Т
680335.9700	3083372.0200	G-56SD	2750	Manual	Т
680022.0080	3083237.4720	J-44SD	1205	Manual	Т
680047.0950	3083215.9420	J-45SD	1960	Manual	Т
680108.0130	3083101.3520	J-57SD	14500	Manual	Т
679958.5608	3083562.3366	G-30SD	2010	Random	

	Area: Area 2								
X Coord	Y Coord	Label	Value	Туре	Historical				
679619.1240	3082932.8980	G-30SD	2010	Manual	Т				
679693.4050	3083047.5490	G-31SD	1670	Manual	Т				
679768.2260	3083162.7270	G-32SD	2860	Manual	Т				
679756.1090	3082922.9390	G-39SD	2060	Manual	Т				
679832.1580	3083041.8710	G-40SD	953	Manual	Т				
679906.4210	3083156.7540	G-41SD	2860	Manual	Т				
679822.0740	3082799.5750	G-48SD	10900	Manual	Т				
679896.6120	3082915.6660	G-49SD	2330	Manual	Т				
679971.2150	3083030.7920	G-50SD	1180	Manual	Т				
679887.4330	3082812.9360	J-46SD	5590	Manual	Т				
679763.3990	3083050.0550	J-56SD	2180	Manual	Т				

	Area: Area 3									
X Coord	Y Coord	Label	Value	Туре	Historical					
679393.4380	3082582.9470	G-27SD	2390	Manual	Т					
679470.2860	3082698.0210	G-28SD	6165	Manual	Т					
679543.3140	3082816.1060	G-29SD	1490	Manual	Т					
679530.9430	3082575.4480	G-36SD	11700	Manual	Т					
679606.6840	3082692.3830	G-37SD	10300	Manual	Т					
679681.4650	3082807.7990	G-38SD	5620	Manual	Т					
679597.1880	3082450.9230	G-45SD	4290	Manual	Т					
679671.3170	3082565.9250	G-46SD	23100	Manual	Т					
679745.9820	3082681.3860	G-47SD	10400	Manual	Т					
679521.7790	3082672.0220	J-54SD	18900	Manual	Т					
679590.0920	3082773.1840	J-55SD	4655	Manual	Т					

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this

site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

### **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

S

n is the number of samples,

is the estimated standard deviation of the measured values including analytical error,

 $\Lambda$  is the width of the gray region.

 $\alpha$  is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

β is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analyte	_			Parar	nete	er	
Analyte	"	S	Δ	α	β	$Z_{1-\alpha}$ a	<b>Z<sub>1-β</sub></b> b
	15	8087.8	6513.2	0.05	0.1	1.64485	1.28155

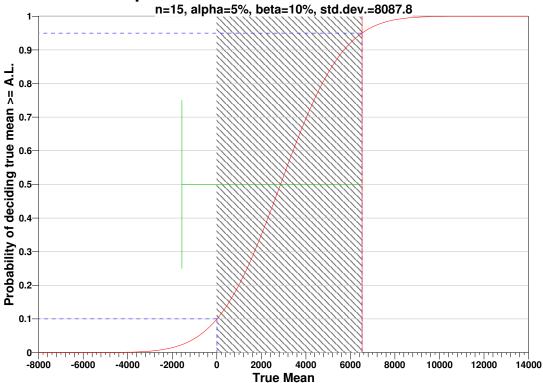
<sup>&</sup>lt;sup>a</sup> This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at 1- $\alpha$ . If any of the inputs change, the number of samples that result in the correct curve changes.

<sup>&</sup>lt;sup>b</sup> This value is automatically calculated by VSP based upon the user defined value of β.

# 1-Sample t-Test of True Mean vs. Action Level



### **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

#### Sensitivity Analysis

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples							
AL=652	1 0	α=	:5	α=10		α=15	
AL=032	1.2	s=16175.6	s=8087.8	s=16175.6 s=8087.8		s=16175.6	s=8087.8
	β=5	6660	1666	5270	1319	4424	1107
LBGR=90	β=10	5271	1319	4043	1012	3307	828
	β=15	4425	1108	3307	828	2645	662
	β=5	1666	418	1319	331	1107	277
LBGR=80	β=10	1319	331	1012	254	828	208
	β=15	1108	278	828	208	662	166
LBGR=70	β=5	742	187	587	148	493	124

β=10	587	148	450	114	368	93
β=15	493	125	369	93	295	74

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

# **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$8,500.00, which averages out to a per sample cost of \$566.67. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION								
Cost Details	Per Analysis	Per Sample	15 Samples					
Field collection costs		\$100.00	\$1,500.00					
Analytical costs	\$400.00	\$400.00	\$6,000.00					
Sum of Field & Analytical costs		\$500.00	\$7,500.00					
Fixed planning and validation costs			\$1,000.00					
Total cost			\$8,500.00					

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	953	971	1080	1180	1205	1490	1670	1790	1900	1960
10	2010	2010	2060	2060	2180	2220	2330	2390	2600	2750
20	2860	2860	2870	3810	4290	4560	4655	5000	5490	5590
30	5620	6165	6525	1.03e+004	1.04e+004	1.09e+004	1.17e+004	1.23e+004	1.34e+004	1.45e+004
40	1.89e+004	2.31e+004	3.47e+004	3.59e+004						

SUMMARY STATISTICS		
n	44	
Min	953	
Max	35900	
Range	34947	
Mean	6663.7	
Median	2865	
Variance	6.5412e+007	
StdDev	8087.8	
Std Error	1219.3	
Skewness	2.3955	
Interquartile Range	7346.3	

	Percentiles							
1%	5%	10%	25%	50%	75%	90%	95%	99%
953	998.3	1193	2010	2865	9356	1.67e+004	3.18e+004	3.59e+004

#### **Outlier Test**

Rosner's test for multiple outliers was performed to test whether the most extreme value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

In using Rosner's test to detect up to 1 outlier, a test statistic  $R_1$  is calculated, and compared with a critical value  $C_1$  to test the hypothesis that there is one outlier in the data.

ROSNER'S OUTLIER TEST				
k	Test Statistic R <sub>k</sub>	5% Critical Value C <sub>k</sub>	Significant?	
1	3.615	3.08	Yes	

The test statistic 3.615 exceeded the corresponding critical value, therefore that test is significant and we conclude that the most extreme value is an outlier at the 5% significance level.

SUSPECTED OUTLIERS			
1	35900		

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Rosner's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding out				
Shapiro-Wilk Test Statistic	0.7062			
Shapiro-Wilk 5% Critical Value	0.943			

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the most extreme value, do not appear to follow a normal distribution at the 5% level of significance. Rosner's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

### **Data Plots**

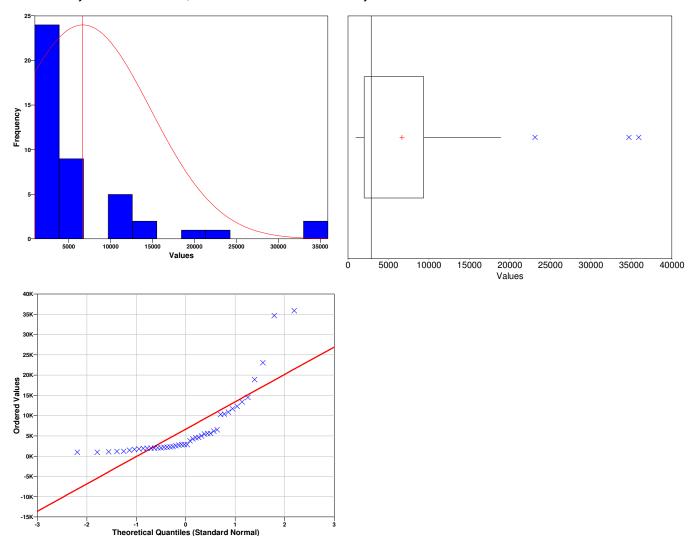
Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5

times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

#### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST				
Shapiro-Wilk Test Statistic	0.6759			
Shapiro-Wilk 5% Critical Value	0.944			

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the

data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE MEAN	
95% Parametric UCL	8713
95% Non-Parametric (Chebyshev) UCL	1.198e+004

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (1.198e+004) may be a more accurate upper confidence limit on the true mean.

#### **One-Sample t-Test**

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\bar{x} - AL}{SE}$$

where

x is the sample mean of the n=44 data,

AL is the action level or threshold (6521.2),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=43 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST				
t-statistic	Critical Value t <sub>0.95</sub>	<b>Null Hypothesis</b>		
0.1169	1.6811	Reject		

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test				
Test Statistic (S+)	95% Critical Value	<b>Null Hypothesis</b>		
32	27	Reject		

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Area: Area 1 X Coord Y Coord Label Value Type Historical G-21SD 971 Manual 679638.466 3083412.561 679715.315 G-22SD 34700 Manual 3083530.019 679789.831 3083644.936 G-23SD 35900 Manual 679780.111 3083404.025 G-24SD 2600 Manual 679854.225 3083519.822 G-25SD 5490 Manual 679931.397 3083636.406 G-26SD 6525 Manual 679393.438 3082582.947 G-27SD 2390 Manual 679470.286 3082698.021 G-28SD 6165 Manual 679543.314 3082816.106 G-29SD 1490 Manual 3082932.898 679619.124 G-30SD 2010 Manual 3083047.549 G-31SD 1670 679693.405 Manual 679768.226 3083162.727 G-32SD 2860 Manual 679841.774 3083280.235 G-33SD 3810 Manual 679917.766 3083397.416 G-34SD 4560 Manual 679994.207 3083513.447 G-35SD 2220 Manual 679530.943 G-36SD 11700 3082575.448 Manual 679606.684 3082692.383 G-37SD 10300 Manual G-38SD 5620 679681.465 3082807.799 Manual 679756.109 G-39SD 2060 Manual 3082922.939 G-40SD 953 Manual 679832.158 3083041.871 679906.421 3083156.754 G-41SD 2860 Manual 679982.277 3083273.065 G-42SD 1080 Manual G-43SD 13400 680056.981 3083387.33 Manual 680132.65 3083505.456 G-44SD 1790 Manual 679597.188 3082450.923 G-45SD 4290 Manual 679671.317 G-46SD 23100 Manual 3082565.925 3082681.386 G-47SD 10400 Manual 679745.982 679822.074 G-48SD 10900 Manual 3082799.575 3082915.666 G-49SD 2330 679896.612 Manual 679971.215 3083030.792 G-50SD 1180 Manual 680046.783 3083146.999 G-51SD 2060 Manual 680123.432 3083263.001 G-52SD 1900 Manual 680198.058 3083379.815 G-53SD 5000 Manual G-54SD 2870 680185.412 3083138.847 Manual 680260.421 3083254.807 G-55SD 12300 Manual 680335.97 3083372.02 G-56SD 2750 Manual J-44SD Manual 680022.008 3083237.472 1205 J-45SD 680047.095 3083215.942 1960 Manual 679887.433 3082812.936 J-46SD 5590 Manual 3082672.022 J-54SD 679521.779 18900 Manual 679590.092 3082773.184 J-55SD 4655 Manual 679763.399 3083050.055 J-56SD 2180 Manual 14500 680108.013 3083101.352 J-57SD Manual 3083297.013 J-58SD 680413.831 2240 Manual

X Coord Y 0	Coord Label	Value	Type	Historical
679530.943	3082575.448	G-36SD	7 1	Manual
679606.684	3082692.383	G-37SD	2.9	Manual
679671.317	3082565.925	G-46SD	1.35 I	Manual
679745.982	3082681.386	G-47SD	1.35 I	Manual
679521.779	3082672.022	J-54SD	1.35 I	Manual
680108.013	3083101.352	J-57SD	10.6 I	Manual
680413.831	3083297.013	J-58SD	1.35 I	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

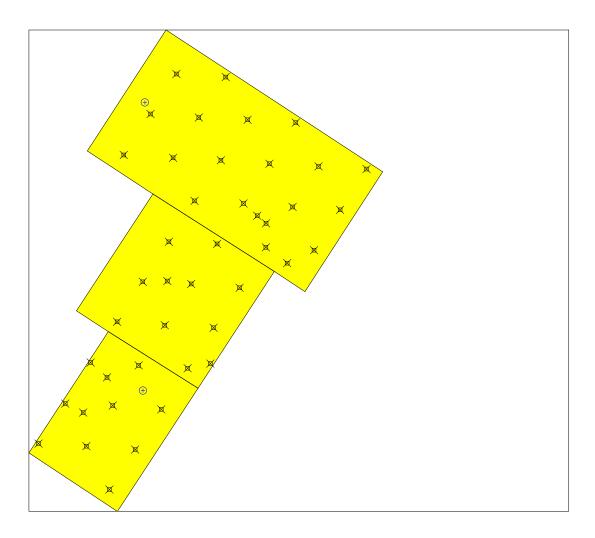
SUMMARY OF	SAMPLING DESIGN
Primary Objective of Design	Compare a site mean to a fixed threshold
Type of Sampling Design	Parametric
Sample Placement (Location) in the Field	Simple random sampling
Working (Null) Hypothesis	The mean value at the site exceeds the threshold
Formula for calculating number of sampling locations	Student's t-test
Calculated total number of samples	21
Number of samples on map <sup>a</sup>	45
Number of selected sample areas b	3
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>
Total cost of sampling <sup>d</sup>	\$11,500.00

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1								
X Coord	Y Coord	Label	Value	Туре	Historical			
679638.4660	3083412.5610	G-21SD	0.33	Manual	Т			
679715.3150	3083530.0190	G-22SD	17.3	Manual	Т			
679789.8310	3083644.9360	G-23SD	6.3	Manual	Т			
679780.1110	3083404.0250	G-24SD	1.4	Manual	Т			
679854.2250	3083519.8220	G-25SD	2.4	Manual	Т			
679931.3970	3083636.4060	G-26SD	2.13	Manual	Т			
679841.7740	3083280.2350	G-33SD	2.8	Manual	Т			
679917.7660	3083397.4160	G-34SD	0.86	Manual	Т			
679994.2070	3083513.4470	G-35SD	1.4	Manual	Т			
679982.2770	3083273.0650	G-42SD	0.43	Manual	Т			
680056.9810	3083387.3300	G-43SD	6.3	Manual	Т			
680132.6500	3083505.4560	G-44SD	1.7	Manual	Т			
680046.7830	3083146.9990	G-51SD	0.625	Manual	Т			
680123.4320	3083263.0010	G-52SD	0.75	Manual	Т			
680198.0580	3083379.8150	G-53SD	1.5	Manual	Т			
680185.4120	3083138.8470	G-54SD	2.8	Manual	Т			

680260.4210	3083254.8070	G-55SD	4.8	Manual	Т
680335.9700	3083372.0200	G-56SD	1.4	Manual	Т
680022.0080	3083237.4720	J-44SD	0.455	Manual	Т
680047.0950	3083215.9420	J-45SD	0.74	Manual	Т
680108.0130	3083101.3520	J-57SD	6.5	Manual	Т
679699.2972	3083563.1712	G-30SD	1.6	Random	

	Area: Area 2								
X Coord	Y Coord	Label	Value	Туре	Historical				
679619.1240	3082932.8980	G-30SD	1.6	Manual	Т				
679693.4050	3083047.5490	G-31SD	1.3	Manual	Т				
679768.2260	3083162.7270	G-32SD	2.2	Manual	Т				
679756.1090	3082922.9390	G-39SD	0.45	Manual	Т				
679832.1580	3083041.8710	G-40SD	0.75	Manual	Т				
679906.4210	3083156.7540	G-41SD	1.5	Manual	Т				
679822.0740	3082799.5750	G-48SD	2.3	Manual	Т				
679896.6120	3082915.6660	G-49SD	0.79	Manual	Т				
679971.2150	3083030.7920	G-50SD	0.31	Manual	Т				
679887.4330	3082812.9360	J-46SD	1.1	Manual	Т				
679763.3990	3083050.0550	J-56SD	1.5	Manual	Т				

	Area: Area 3									
X Coord	Y Coord	Label	Value	Туре	Historical					
679393.4380	3082582.9470	G-27SD	1.6	Manual	Т					
679470.2860	3082698.0210	G-28SD	1.6	Manual	Т					
679543.3140	3082816.1060	G-29SD	1.7	Manual	Т					
679530.9430	3082575.4480	G-36SD	3.3	Manual	Т					
679606.6840	3082692.3830	G-37SD	4.7	Manual	Т					
679681.4650	3082807.7990	G-38SD	2.4	Manual	Т					
679597.1880	3082450.9230	G-45SD	0.67	Manual	Т					
679671.3170	3082565.9250	G-46SD	8.9	Manual	Т					
679745.9820	3082681.3860	G-47SD	2.6	Manual	Т					
679521.7790	3082672.0220	J-54SD	5	Manual	Т					
679590.0920	3082773.1840	J-55SD	1.3	Manual	Т					
679693.6062	3082734.8361		0	Random						

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations.

A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

### **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis if the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

*n* is the number of samples,

is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

α is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

β is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analysta	_				Para	ameter	
Analyte	11	$S \Delta \alpha \beta Z_{1-\alpha}^{a} Z_{1-\beta}^{b}$					<b>Z</b> <sub>1-β</sub> b
	21	3	2	0.05	0.1	1.64485	1.28155

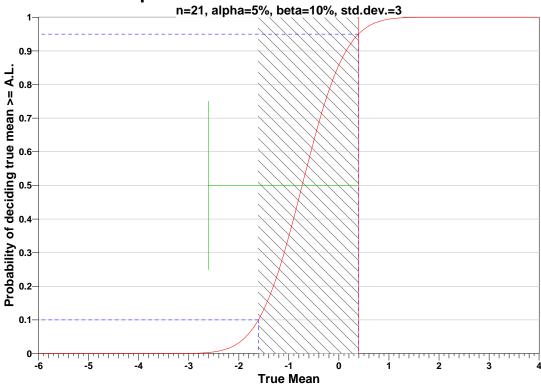
 $^{\text{a}}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ 

<sup>b</sup> This value is automatically calculated by VSP based upon the user defined value of β.

The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at 1- $\alpha$ . If any of the inputs change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



## **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

### **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

	Number of Samples								
A1 _0.3		α=	5	α=	10	α=′	α=15		
AL=0.3	9	s=6	s=3	s=6 s=3		s=6	s=3		
	β=5	256148	64038	202696	50675	170162	42541		
LBGR=90	β=10	202696	50676	155492	38874	127174	31794		
	β=15	170163	42542	127174	31795	101700	25426		
	β=5	64038	16011	50675	12670	42541	10636		
LBGR=80	β=10	50676	12670	38874	9720	31794	7949		
	β=15	42542	10637	31795	7950	25426	6357		
LBGR=70	β=5	28463	7117	22523	5632	18908	4728		

β=10	22523	5632	17278	4321	14131	3534
β=15	18909	4729	14132	3534	11301	2826

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

# **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$11,500.00, which averages out to a per sample cost of \$547.62. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION									
Cost Details	Per Analysis	Per Sample	21 Samples						
Field collection costs		\$100.00	\$2,100.00						
Analytical costs	\$400.00	\$400.00	\$8,400.00						
Sum of Field & Analytical costs		\$500.00	\$10,500.00						
Fixed planning and validation costs			\$1,000.00						
Total cost			\$11,500.00						

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	0	0	0.31	0.33	0.43	0.45	0.455	0.625	0.67	0.74
10	0.75	0.75	0.79	0.86	1.1	1.3	1.3	1.4	1.4	1.4
20	1.5	1.5	1.5	1.6	1.6	1.6	1.6	1.7	1.7	2.13
30	2.2	2.3	2.4	2.4	2.6	2.8	2.8	3.3	4.7	4.8
40	5	6.3	6.3	6.5	8.9	17.3				

SUMMARY STATISTICS						
n	46					
Min	0					
Max	17.3					
Range	17.3					
Mean	2.4367					
Median	1.55					
Variance	8.768					
StdDev	2.9611					
Std Error	0.43659					
Skewness	3.2734					
Interquartile Range	1.9					

	Percentiles							
1%	1% 5% 10% 25% 50% 75% 90% 95% 99%							99%
0	0.1085	0.4	0.75	1.55	2.65	6.3	8.06	17.3

#### **Outlier Test**

Rosner's test for multiple outliers was performed to test whether the most extreme value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

In using Rosner's test to detect up to 1 outlier, a test statistic  $R_1$  is calculated, and compared with a critical value  $C_1$  to test the hypothesis that there is one outlier in the data.

ROSNER'S OUTLIER TEST								
k Test Statistic R <sub>k</sub> 5% Critical Value C <sub>k</sub> Significant?								
1	4.962	3.09	Yes					

The test statistic 4.962 exceeded the corresponding critical value, therefore that test is significant and we conclude that the most extreme value is an outlier at the 5% significance level.

SUSPECTED	OUTLIERS
1	17.3

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Rosner's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)				
Shapiro-Wilk Test Statistic	0.8116			
Shapiro-Wilk 5% Critical Value	0.944			

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the most extreme value, do not appear to follow a normal distribution at the 5% level of significance. Rosner's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

### **Data Plots**

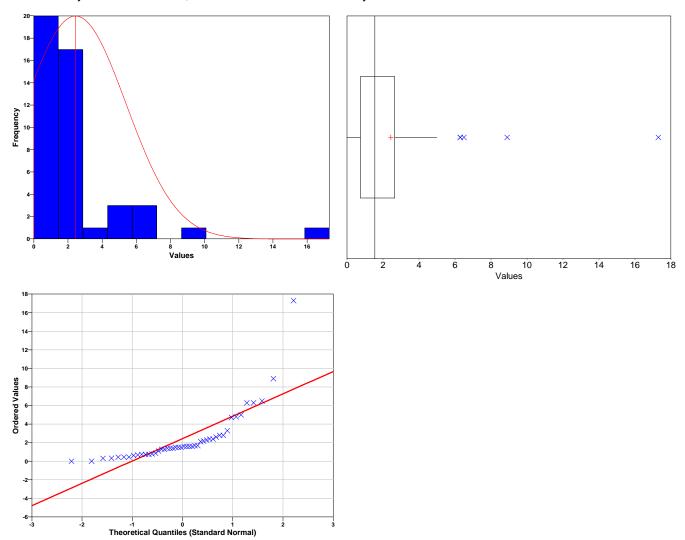
Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5

times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

#### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST			
Shapiro-Wilk Test Statistic	0.6649		
Shapiro-Wilk 5% Critical Value	0.945		

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the

data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE MEAN			
95% Parametric UCL	3.17		
95% Non-Parametric (Chebyshev) UCL	4.34		

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (4.34) may be a more accurate upper confidence limit on the true mean.

### One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\bar{x} - AL}{SE}$$

#### where

x is the sample mean of the n=46 data,

AL is the action level or threshold (0.39),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=45 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST			
t-statistic Critical Value $t_{0.95}$ Null Hypothesis			
4.688	1.6794	Reject	

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test					
Test Statistic (S+) 95% Critical Value Null Hypothesis					
4	29	Cannot Reject			

Note: There may not be enough data to reject the null hypothesis (and conclude site is clean) with 95% confidence using the MARSSIM sign test.

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Software and documentation available at http://dqo.pnl.gov/vsp

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Area: Area 1 X Coord Y Coord Label Value Type Historical G-21SD 0.33 Manual 679638.466 3083412.561 679715.315 3083530.019 G-22SD 17.3 Manual 679789.831 3083644.936 G-23SD 6.3 Manual 679780.111 3083404.025 G-24SD 1.4 Manual 3083519.822 G-25SD 2.4 Manual 679854.225 679931.397 3083636.406 G-26SD 2.13 Manual 679393.438 3082582.947 G-27SD 1.6 Manual 679470.286 3082698.021 G-28SD 1.6 Manual 679543.314 3082816.106 G-29SD 1.7 Manual 679619.124 3082932.898 G-30SD 1.6 Manual 3083047.549 G-31SD 1.3 Manual 679693.405 679768.226 3083162.727 G-32SD 2.2 Manual 679841.774 3083280.235 G-33SD 2.8 Manual 679917.766 3083397.416 G-34SD 0.86 Manual 679994.207 3083513.447 G-35SD 1.4 Manual 3082575.448 G-36SD 3.3 Manual 679530.943 679606.684 3082692.383 G-37SD 4.7 Manual 3082807.799 G-38SD 2.4 Manual 679681.465 3082922.939 G-39SD 0.45 Manual 679756.109 679832.158 3083041.871 G-40SD 0.75 Manual 679906.421 3083156.754 G-41SD 1.5 Manual 679982.277 3083273.065 G-42SD 0.43 Manual G-43SD 6.3 Manual 680056.981 3083387.33 680132.65 3083505.456 G-44SD 1.7 Manual 679597.188 3082450.923 G-45SD 0.67 Manual 3082565.925 G-46SD 8.9 Manual 679671.317 3082681.386 G-47SD 2.6 Manual 679745.982 3082799.575 G-48SD 2.3 Manual 679822.074 3082915.666 G-49SD 0.79 Manual 679896.612 679971.215 3083030.792 G-50SD 0.31 Manual 680046.783 3083146.999 G-51SD 0.625 Manual 680123.432 3083263.001 G-52SD 0.75 Manual 680198.058 3083379.815 G-53SD 1.5 Manual 3083138.847 G-54SD 2.8 Manual 680185.412 680260.421 3083254.807 G-55SD 4.8 Manual 680335.97 3083372.02 G-56SD 1.4 Manual J-44SD 680022.008 3083237.472 0.455 Manual 3083215.942 J-45SD 680047.095 0.74 Manual 679887.433 3082812.936 J-46SD 1.1 Manual 3082672.022 J-54SD 679521.779 5 Manual 679590.092 3082773.184 J-55SD 1.3 Manual 3083050.055 J-56SD 1.5 Manual 679763.399 680108.013 3083101.352 J-57SD 6.5 Manual

3083297.013 J-58SD

0.86 Manual

680413.831

X Coord Y C	Coord Label	Value	Type	Historical
679924.815	3082872.349	J-47S	1.3 M	anual
679994.969	3082983.51	J-48S	1.2 M	anual
680057.658	3083072.075	J-49S	0.69 M	anual
680077.354	3083115.533	J-50S	0.52 M	anual
679827.115	3082729.746	J-51S	0.63 M	anual
680141.873	3083080.88	J-52S	1.1 M	anual
680170.56	3083064.674	J-53S	2.4 M	anual

X Coord Y	Coord Label	Value	Type	Historical
679924.815	3082872.349	J-47S	1.1 Ma	ınual
679994.969	3082983.51	J-48S	0.965	Manual
680057.658	3083072.075	J-49S	1 <b>M</b> a	ınual
680077.354	3083115.533	J-50S	0.72 Ma	ınual
679827.115	3082729.746	J-51S	0.96 Ma	ınual
680141.873	3083080.88	J-52S	1 <b>M</b> a	ınual
680170.56	3083064.674	J-53S	2.5 Ma	ınual

X Coord Y C	oord Label	Value	Type	Historical
679924.815	3082872.349	J-47S	0.086	Manual
679994.969	3082983.51	J-48S	0.087	Manual
680057.658	3083072.075	J-49S	0.084	Manual
680077.354	3083115.533	J-50S	0.088	Manual
679827.115	3082729.746	J-51S	0.108	Manual
680141.873	3083080.88	J-52S	0.1 Mai	nual
680170.56	3083064.674	J-53S	0.086	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

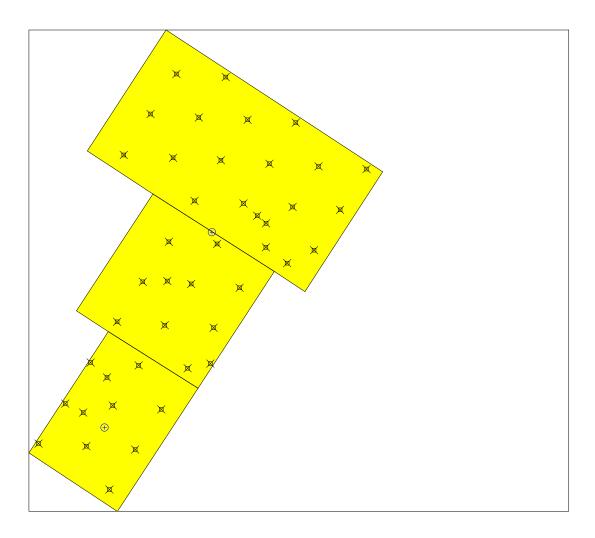
SUMMARY OF	SAMPLING DESIGN
Primary Objective of Design	Compare a site mean to a fixed threshold
Type of Sampling Design	Parametric
Sample Placement (Location) in the Field	Simple random sampling
Working (Null) Hypothesis	The mean value at the site exceeds the threshold
Formula for calculating number of sampling locations	Student's t-test
Calculated total number of samples	21
Number of samples on map <sup>a</sup>	45
Number of selected sample areas b	3
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>
Total cost of sampling <sup>d</sup>	\$11,500.00

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1					
X Coord	Y Coord	Label	Value	Туре	Historical
679638.4660	3083412.5610	G-21SD	0.153	Manual	Т
679715.3150	3083530.0190	G-22SD	0.215	Manual	Т
679789.8310	3083644.9360	G-23SD	0.075	Manual	Т
679780.1110	3083404.0250	G-24SD	0.05	Manual	Т
679854.2250	3083519.8220	G-25SD	0.06	Manual	Т
679931.3970	3083636.4060	G-26SD	0.0525	Manual	Т
679841.7740	3083280.2350	G-33SD	0.055	Manual	Т
679917.7660	3083397.4160	G-34SD	0.0495	Manual	Т
679994.2070	3083513.4470	G-35SD	0.05	Manual	Т
679982.2770	3083273.0650	G-42SD	0.046	Manual	Т
680056.9810	3083387.3300	G-43SD	0.729	Manual	Т
680132.6500	3083505.4560	G-44SD	0.055	Manual	Т
680046.7830	3083146.9990	G-51SD	0.068	Manual	Т
680123.4320	3083263.0010	G-52SD	0.047	Manual	Т
680198.0580	3083379.8150	G-53SD	0.055	Manual	Т
680185.4120	3083138.8470	G-54SD	0.055	Manual	Т

680260.4210	3083254.8070	G-55SD	0.085	Manual	Т
680335.9700	3083372.0200	G-56SD	0.0495	Manual	Т
680022.0080	3083237.4720	J-44SD	0.04825	Manual	Т
680047.0950	3083215.9420	J-45SD	0.049	Manual	Т
680108.0130	3083101.3520	J-57SD	0.065	Manual	Т

	Area: Area 2								
X Coord	Y Coord	Label	Value	Туре	Historical				
679619.1240	3082932.8980	G-30SD	0.0485	Manual	Т				
679693.4050	3083047.5490	G-31SD	0.048	Manual	Т				
679768.2260	3083162.7270	G-32SD	0.055	Manual	Т				
679756.1090	3082922.9390	G-39SD	0.055	Manual	Т				
679832.1580	3083041.8710	G-40SD	0.0485	Manual	Т				
679906.4210	3083156.7540	G-41SD	0.06	Manual	Т				
679822.0740	3082799.5750	G-48SD	0.065	Manual	Т				
679896.6120	3082915.6660	G-49SD	0.04725	Manual	Т				
679971.2150	3083030.7920	G-50SD	0.0465	Manual	Т				
679887.4330	3082812.9360	J-46SD	0.05	Manual	Т				
679763.3990	3083050.0550	J-56SD	0.05	Manual	Т				
679891.8976	3083190.4325	G-27SD	0.0485	Random					

	Area: Area 3								
X Coord	Y Coord	Label	Value	Туре	Historical				
679393.4380	3082582.9470	G-27SD	0.0485	Manual	Т				
679470.2860	3082698.0210	G-28SD	0.04975	Manual	Т				
679543.3140	3082816.1060	G-29SD	0.444	Manual	Т				
679530.9430	3082575.4480	G-36SD	0.065	Manual	Т				
679606.6840	3082692.3830	G-37SD	0.095	Manual	Т				
679681.4650	3082807.7990	G-38SD	0.055	Manual	Т				
679597.1880	3082450.9230	G-45SD	0.408	Manual	Т				
679671.3170	3082565.9250	G-46SD	0.342	Manual	Т				
679745.9820	3082681.3860	G-47SD	0.065	Manual	Т				
679521.7790	3082672.0220	J-54SD	0.1	Manual	Т				
679590.0920	3082773.1840	J-55SD	0.055	Manual	Т				
679583.0667	3082628.6712		0	Random					

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations.

A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

### **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis if the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

*n* is the number of samples,

is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

α is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

β is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analysta	_				Para	ameter	
Analyte	11	S	Δ	α	β	<b>Z</b> <sub>1-α</sub> a	<b>Z</b> <sub>1-β</sub> b
	21	3	2	0.05	0.1	1.64485	1.28155

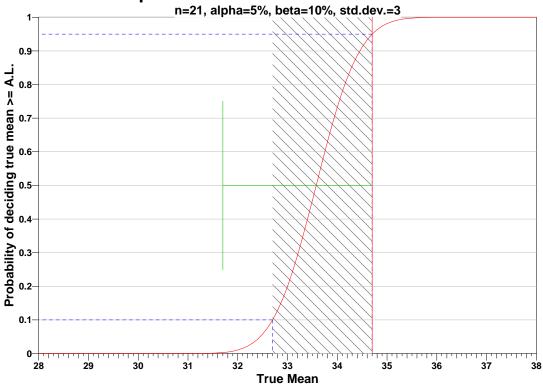
 $^{\text{a}}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ 

<sup>b</sup> This value is automatically calculated by VSP based upon the user defined value of β.

The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at 1- $\alpha$ . If any of the inputs change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



# **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples									
A1 -24	7	α=	=5	α=	α=10		α=15		
AL=34	.1	s=6	s=3	s=6	s=3	s=6	s=3		
	β=5	34	10	27	8	23	6		
LBGR=90	β=10	27	8	21	6	17	5		
	β=15	23	7	17	5	14	4		
	β=5	10	4	8	3	6	2		
LBGR=80	β=10	8	3	6	3	5	2		
	β=15	7	3	5	2	4	2		
LBGR=70	β=5	5	3	4	2	3	2		

β=10	5	3	4	2	3	1
β=15	4	2	3	2	2	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

# **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$11,500.00, which averages out to a per sample cost of \$547.62. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION								
Cost Details	Per Analysis	Per Sample	21 Samples					
Field collection costs		\$100.00	\$2,100.00					
Analytical costs	\$400.00	\$400.00	\$8,400.00					
Sum of Field & Analytical costs		\$500.00	\$10,500.00					
Fixed planning and validation costs			\$1,000.00					
Total cost			\$11,500.00					

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	0	0	0.046	0.0465	0.047	0.04725	0.048	0.04825	0.0485	0.0485
10	0.0485	0.0485	0.049	0.0495	0.0495	0.04975	0.05	0.05	0.05	0.05
20	0.0525	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.055	0.06
30	0.06	0.065	0.065	0.065	0.065	0.068	0.075	0.085	0.095	0.1
40	0.153	0.215	0.342	0.408	0.444	0.729				

SUMMARY ST	SUMMARY STATISTICS					
n	46					
Min	0					
Max	0.729					
Range	0.729					
Mean	0.096984					
Median	0.055					
Variance	0.017313					
StdDev	0.13158					
Std Error	0.0194					
Skewness	3.4272					
Interquartile Range	0.01725					

	Percentiles							
1%	5%	10%	25%	50%	75%	90%	95%	99%
0	0.0161	0.04685	0.0485	0.055	0.06575	0.2531	0.4314	0.729

#### **Outlier Test**

Rosner's test for multiple outliers was performed to test whether the most extreme value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

In using Rosner's test to detect up to 1 outlier, a test statistic  $R_1$  is calculated, and compared with a critical value  $C_1$  to test the hypothesis that there is one outlier in the data.

ROSNER'S OUTLIER TEST						
k	Test Statistic R <sub>k</sub>	5% Critical Value C <sub>k</sub>	Significant?			
1	4.749	3.09	Yes			

The test statistic 4.749 exceeded the corresponding critical value, therefore that test is significant and we conclude that the most extreme value is an outlier at the 5% significance level.

SUSPECTED OUTLIERS			
1	0.729		

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Rosner's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)						
Shapiro-Wilk Test Statistic	0.521					
Shapiro-Wilk 5% Critical Value	0.944					

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the most extreme value, do not appear to follow a normal distribution at the 5% level of significance. Rosner's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

### **Data Plots**

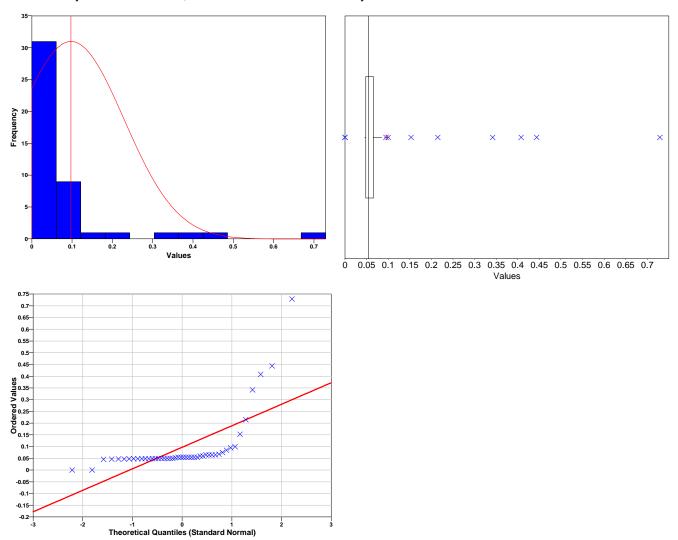
Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5

times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

#### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST	
Shapiro-Wilk Test Statistic	0.4953
Shapiro-Wilk 5% Critical Value	0.945

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the

data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

## **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLS ON THE MEAN					
95% Parametric UCL	0.1296				
95% Non-Parametric (Chebyshev) UCL	0.1815				

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (0.1815) may be a more accurate upper confidence limit on the true mean.

# One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\overline{x} - AL}{SE}$$

where

x is the sample mean of the n=46 data,

AL is the action level or threshold (34.7),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=45 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST						
t-statistic	Critical Value t <sub>0.95</sub>	<b>Null Hypothesis</b>				
-1783.7	1.6794	Reject				

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test								
Test Statistic (S+)	95% Critical Value	<b>Null Hypothesis</b>						
46	29	Reject						

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Area: Area 1 X Coord Y Coord Label Value Type Historical 679638.466 3083412.561 G-21SD 0.153 Manual 679715.315 3083530.019 G-22SD 0.215 Manual 679789.831 3083644.936 G-23SD 0.075 Manual 679780.111 3083404.025 G-24SD 0.05 Manual 3083519.822 G-25SD 0.06 Manual 679854.225 3083636.406 G-26SD 0.0525 679931.397 Manual 679393.438 3082582.947 G-27SD 0.0485 Manual 679470.286 3082698.021 G-28SD 0.04975 Manual 679543.314 3082816.106 G-29SD 0.444 Manual 3082932.898 679619.124 G-30SD 0.0485 Manual 3083047.549 G-31SD 0.048 679693.405 Manual 679768.226 3083162.727 G-32SD 0.055 Manual 679841.774 3083280.235 G-33SD 0.055 Manual 679917.766 3083397.416 G-34SD 0.0495 Manual 679994.207 3083513.447 G-35SD 0.05 Manual 679530.943 3082575.448 G-36SD 0.065 Manual 679606.684 3082692.383 G-37SD 0.095 Manual G-38SD 0.055 679681.465 3082807.799 Manual 679756.109 3082922.939 G-39SD 0.055 Manual 679832.158 3083041.871 G-40SD 0.0485 Manual 679906.421 3083156.754 G-41SD 0.06 Manual 679982.277 3083273.065 G-42SD 0.046 Manual G-43SD 0.729 680056.981 3083387.33 Manual 680132.65 3083505.456 G-44SD 0.055 Manual 679597.188 3082450.923 G-45SD 0.408 Manual 3082565.925 G-46SD 0.342 Manual 679671.317 3082681.386 G-47SD 0.065 679745.982 Manual 679822.074 G-48SD 0.065 Manual 3082799.575 3082915.666 G-49SD 0.04725 679896.612 Manual 679971.215 3083030.792 G-50SD 0.0465 Manual 680046.783 3083146.999 G-51SD 0.068 Manual 680123.432 3083263.001 G-52SD 0.047 Manual 680198.058 3083379.815 G-53SD 0.055 Manual G-54SD 0.055 680185.412 3083138.847 Manual 680260.421 3083254.807 G-55SD 0.085 Manual 680335.97 3083372.02 G-56SD 0.0495 Manual J-44SD 0.04825 Manual 680022.008 3083237.472 J-45SD 0.049 680047.095 3083215.942 Manual 679887.433 3082812.936 J-46SD 0.05 Manual 3082672.022 J-54SD 679521.779 0.1 Manual 3082773.184 J-55SD 0.055 679590.092 Manual 679763.399 3083050.055 J-56SD 0.05 Manual 680108.013 3083101.352 J-57SD 0.065 Manual

3083297.013 J-58SD

0.055

Manual

680413.831

Area: Area 1

X Coord Y	Coord Label	Value	Туре	e Historical
679924.815	3082872.349	J-47S	2.7	Manual
679994.969	3082983.51	J-48S	3.4	Manual
680057.658	3083072.075	J-49S	3.6	Manual
680077.354	3083115.533	J-50S	3.2	Manual
679827.115	3082729.746	J-51S	2.2	Manual
680141.873	3083080.88	J-52S	4	Manual
680170.56	3083064.674	J-53S	3.9	Manual

Area: Area 1

X Coord Y	Coord Label	Value	Type	e Historical
679924.815	3082872.349	J-47S	4.8	Manual
679994.969	3082983.51	J-48S	5	Manual
680057.658	3083072.075	J-49S	4.1	Manual
680077.354	3083115.533	J-50S	1.4	Manual
679827.115	3082729.746	J-51S	3.5	Manual
680141.873	3083080.88	J-52S	3.6	Manual
680170.56	3083064.674	J-53S	5.9	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

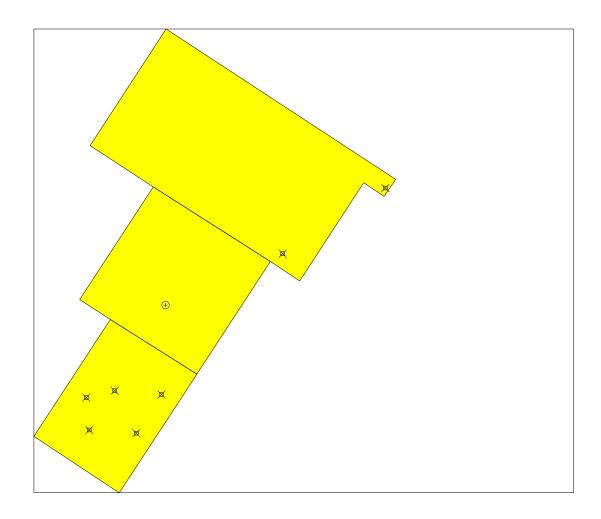
SUMMARY OF	SAMPLING DESIGN
Primary Objective of Design	Compare a site mean to a fixed threshold
Type of Sampling Design	Parametric
Sample Placement (Location) in the Field	Simple random sampling
Working (Null) Hypothesis	The mean value at the site exceeds the threshold
Formula for calculating number of sampling locations	Student's t-test
Calculated total number of samples	2
Number of samples on map <sup>a</sup>	8
Number of selected sample areas b	3
Specified sampling area <sup>c</sup>	606651.66 m <sup>2</sup>
Total cost of sampling d	\$2,000.00

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1								
X Coord Y Coord		Label	Value	Туре	Historical			
680108.0130	3083101.3520	J-57SD	5.9	Manual	Т			
680413.8310	3083297.0130	J-58SD	5.9	Manual	Т			

Area: Area 2							
X Coord Y Coord		Label	Value	Туре	Historical		
679758.7929	3082947.8296	G-36SD	5.9	Random			

Area: Area 3									
X Coord Y Coord		Label	Value	Туре	Historical				
679530.9430	3082575.4480	G-36SD	5.9	Manual	Т				
679606.6840	3082692.3830	G-37SD	5.9	Manual	Т				
679671.3170	3082565.9250	G-46SD	5.9	Manual	Т				
679745.9820	3082681.3860	G-47SD	5.9	Manual	Т				
679521.7790	3082672.0220	J-54SD	7.7	Manual	Т				

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null

hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

# **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

is the number of samples,

S is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analyta	_			Para	met	er	
Analyte	n	S	Δ	α	β	<b>Z</b> <sub>1-α</sub> a	<b>Z<sub>1-β</sub></b> b
	2	2.2662	1292	0.05	0.1	1.64485	1.28155

 $^{\rm a}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

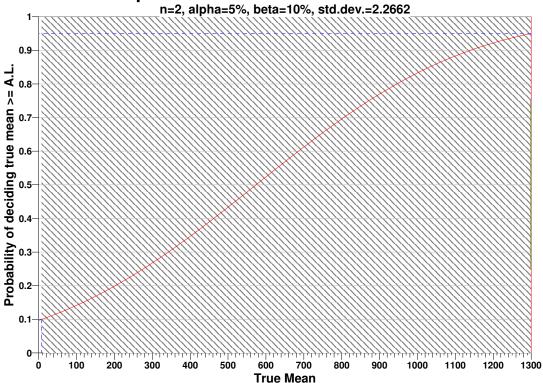
The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at  $1-\alpha$ . If any of the inputs

<sup>&</sup>lt;sup>b</sup> This value is automatically calculated by VSP based upon the user defined value of β.

change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



# **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- 4. the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples									
AL=1300		α=5		α=	:10	α=15			
		s=4.5324	s=2.2662	s=4.5324	s=2.2662	s=4.5324	s=2.2662		
	β=5	2	2	1	1	1	1		
LBGR=90	β=10	2	2	1	1	1	1		
	β=15	2	2	1	1	1	1		
	β=5	2	2	1	1	1	1		
LBGR=80	β=10	2	2	1	1	1	1		
	β=15	2	2	1	1	1	1		

LBGR=70	β=5	2	2	1	1	1	1
	β=10	2	2	1	1	1	1
	β=15	2	2	1	1	1	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

### **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$2,000.00, which averages out to a per sample cost of \$1,000.00. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION					
Cost Details	Per Analysis	Per Sample	2 Samples		
Field collection costs		\$100.00	\$200.00		
Analytical costs	\$400.00	\$400.00	\$800.00		
Sum of Field & Analytical costs		\$500.00	\$1,000.00		
Fixed planning and validation costs			\$1,000.00		
Total cost			\$2,000.00		

#### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	0	5.9	5.9	5.9	5.9	5.9	5.9	5.9	7.7	

SUMMARY				STA	TISTI	cs		
n			9					
Min				0				
	М	ах				7.7		
	Rai	nge				7.7		
	Ме	ean			į	5.4444	1	
Median					5.9			
Variance			4.5228					
StdDev			2.1267					
Std Error				0	.7088	9		
	Skew	ness		-2.4937				
Interquartile Range			0					
Pero				entile	es			
1%	5%	10%	25%	50%	75%	90%	95%	99%
0	0	0	5.9	5.9	5.9	7.7	7.7	7.7

#### **Outlier Test**

Dixon's extreme value test was performed to test whether the lowest value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

DIXON'S OUTLIER TEST				
Dixon Test Statistic	1			
Dixon 5% Critical Value	0.554			

The calculated test statistic exceeds the critical value, so the test rejects the null hypothesis that there are no outliers in the data, and concludes that the minimum value 0 is an outlier at the 5% significance level.

SUSPECTED OUTLIERS			
Min	0		

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Dixon's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)				
Shapiro-Wilk Test Statistic	0.4533			
Shapiro-Wilk 5% Critical Value	0.803			

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the minimum value 0, do not appear to follow a normal distribution at the 5% level of significance. Dixon's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

#### **Data Plots**

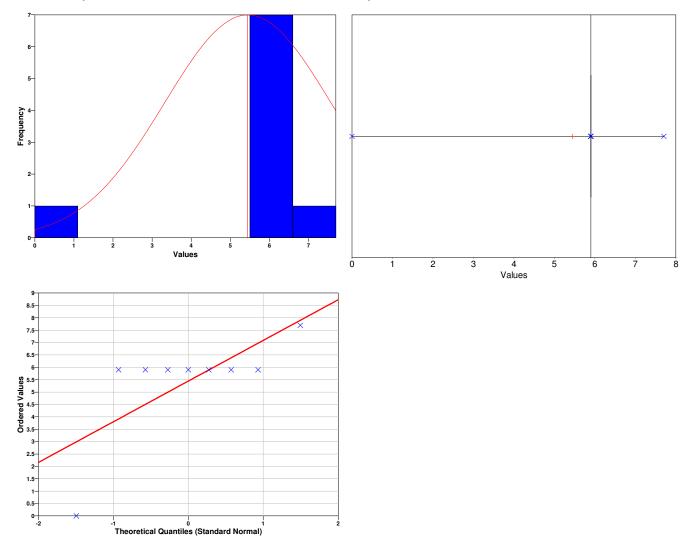
Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5 times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_p$ , for which a

fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/guality/ga-docs.html).

#### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST				
Shapiro-Wilk Test Statistic 0.5681				
Shapiro-Wilk 5% Critical Value	0.829			

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

#### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

**UCLS ON THE MEAN** 

95% Parametric UCL	6.763
95% Non-Parametric (Chebyshev) UCL	8.534

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (8.534) may be a more accurate upper confidence limit on the true mean.

### One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\overline{x} - AL}{SE}$$

# where

x is the sample mean of the n=9 data, AL is the action level or threshold (1300),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=8 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST				
t-statistic Critical Value t <sub>0.95</sub> Null Hypothesis				
-1826.2	1.8595	Reject		

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test				
Test Statistic (S+) 95% Critical Value Null Hypothesis				
9	7	Reject		

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\* - The report contents may have been modified or reformatted by end-user of software.

Area: Area 1

X Coord Y C	Coord Label	Value	Type	e Historical
679530.943	3082575.448	G-36SD	5.9	Manual
679606.684	3082692.383	G-37SD	5.9	Manual
679671.317	3082565.925	G-46SD	5.9	Manual
679745.982	3082681.386	G-47SD	5.9	Manual
679521.779	3082672.022	J-54SD	7.7	Manual
680108.013	3083101.352	J-57SD	5.9	Manual
680413.831	3083297.013	J-58SD	5.9	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

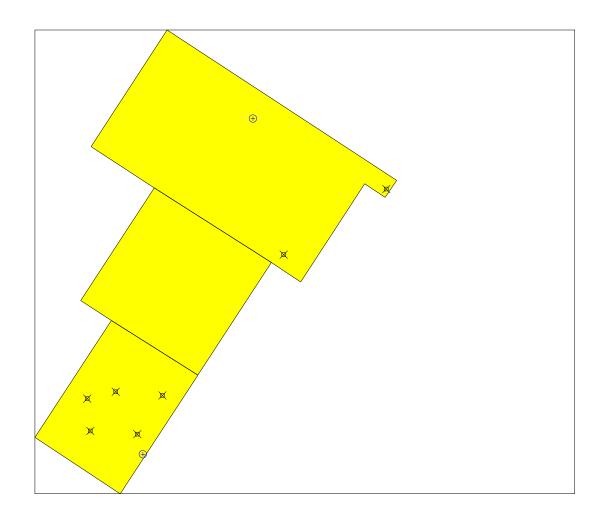
SUMMARY OF SAMPLING DESIGN				
Primary Objective of Design	Compare a site mean to a fixed threshold			
Type of Sampling Design	Parametric			
Sample Placement (Location) in the Field	Simple random sampling			
Working (Null) Hypothesis	The mean value at the site exceeds the threshold			
Formula for calculating number of sampling locations	Student's t-test			
Calculated total number of samples	3			
Number of samples on map <sup>a</sup>	9			
Number of selected sample areas b	3			
Specified sampling area <sup>c</sup>	606651.66 m <sup>2</sup>			
Total cost of sampling <sup>d</sup>	\$2,500.00			

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1										
X Coord	Y Coord	Label	Value	Туре	Historical					
680108.0130	3083101.3520	J-57SD	4.8	Manual	Т					
680413.8310	3083297.0130	J-58SD	10.2	Manual	Т					
680016.1816	3083507.3266	G-36SD	5.3	Random						

Area: Area 2								
X Coord	Y Coord	Label	Value	Туре	Historical			

Area: Area 3									
X Coord	Y Coord	Label	Value	Туре	Historical				
679530.9430	3082575.4480	G-36SD	5.3	Manual	Т				
679606.6840	3082692.3830	G-37SD	4	Manual	Т				
679671.3170	3082565.9250	G-46SD	3.2	Manual	Т				
679745.9820	3082681.3860	G-47SD	1.4	Manual	Т				
679521.7790	3082672.0220	J-54SD	1.525	Manual	Т				
679687.9206	3082505.6009		0	Random					

**Primary Sampling Objective**The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis

(or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

#### **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

# **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

n is the number of samples.

S is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

 $\alpha$  is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analysta	_		Parameter					
Analyte	11	s	Δ	α	β	<b>Z</b> <sub>1-α</sub> a	<b>Z</b> <sub>1-β</sub> b	
	3	3	7	0.05	0.1	1.64485	1.28155	

<sup>a</sup> This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

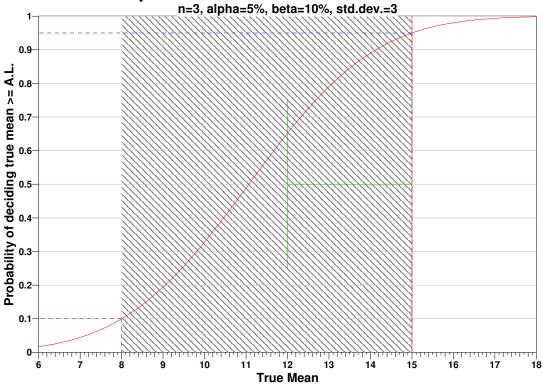
The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the

b This value is automatically calculated by VSP based upon the user defined value of β.

threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at 1- $\alpha$ . If any of the inputs change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



## **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- 4. the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# Sensitivity Analysis

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples								
A1 45		α=5 α=10 α=15						
AL=1	<b>.</b>	s=6	s=3	s=6	s=3	s=6	s=3	
	β=5	175	45	138	36	116	30	
LBGR=90	β=10	139	36	106	28	87	23	
	β=15	117	31	87	23	70	18	
LBGR=80	β=5	45	13	36	10	30	8	
LBGR=00	β=10	36	10	28	8	23	6	

	β=15	31	9	23	7	18	5
	β=5	21	7	17	5	14	4
LBGR=70	β=10	17	6	13	4	11	3
	β=15	15	5	11	4	9	3

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

# **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$2,500.00, which averages out to a per sample cost of \$833.33. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION									
Cost Details	Per Analysis	Per Sample	3 Samples						
Field collection costs		\$100.00	\$300.00						
Analytical costs	\$400.00	\$400.00	\$1,200.00						
Sum of Field & Analytical costs		\$500.00	\$1,500.00						
Fixed planning and validation costs			\$1,000.00						
Total cost			\$2,500.00						

# **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	0	1.4	1.525	3.2	4	4	4.8	5.3	5.3	10.2

		SUM	STA	TISTI	cs				
	ı	n				10			
	М	lin				0			
	М	ах				10.2			
	Ra	nge				10.2			
	Мє	ean			(	3.9725	5		
	Ме	dian		4					
	Vari	ance		7.9976					
	Std	Dev				2.828			
	Std	Error		0.89429					
	Skew	ness		0.93294					
Inte	Interquartile Range				3.8062				
			entile	es					
1%	5%	10%	25%	50%	75%	90%	95%	99%	

0 0 0.14 1.494	4 5.3	9.71 10.2	10.2
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#### **Outlier Test**

Dixon's extreme value test was performed to test whether the lowest value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

DIXON'S OUTLIER TEST						
Dixon Test Statistic 0.032051						
Dixon 5% Critical Value	0.512					

The calculated test statistic does not exceed the critical value, so the test cannot reject the null hypothesis that there are no outliers in the data, and concludes that the minimum value 0 is not an outlier at the 5% significance level.

Because Dixon's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION T	EST (excluding outliers)					
Shapiro-Wilk Test Statistic 0.8653						
Shapiro-Wilk 5% Critical Value 0.818						

The calculated Shapiro-Wilk test statistic exceeds the 5% Shapiro-Wilk critical value, so the test cannot reject the hypothesis that the data are normal and concludes that the data, excluding the minimum value 0, do appear to follow a normal distribution at the 5% level of significance.

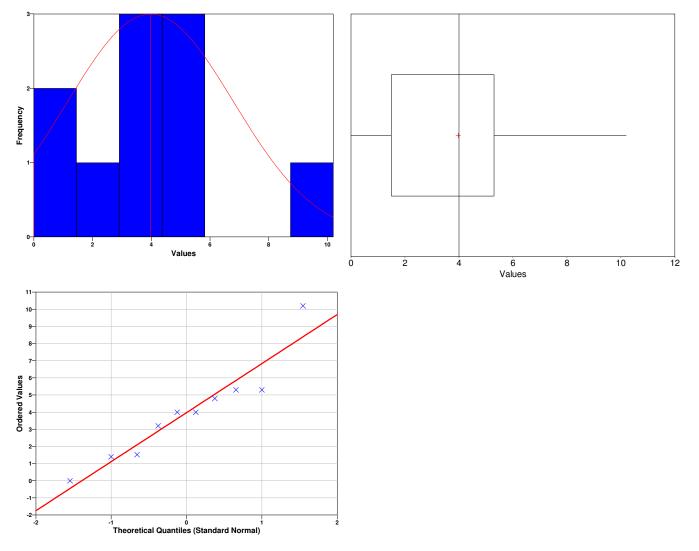
#### **Data Plots**

Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5 times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

#### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST					
Shapiro-Wilk Test Statistic	0.9209				
Shapiro-Wilk 5% Critical Value	0.842				

The calculated SW test statistic exceeds the 5% Shapiro-Wilk critical value, so we cannot reject the hypothesis that the data are normal, or in other words the data appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE MEAN				
95% Parametric UCL	5.612			

Because the data appear to be normally distributed according to the goodness-of-fit test performed above, the parametric UCL (5.612) may be a more accurate upper confidence limit on the true mean.

## One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\bar{x} - AL}{SE}$$

where

x is the sample mean of the n=10 data, AL is the action level or threshold (15),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=9 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST					
t-statistic	Critical Value t <sub>0.95</sub>	<b>Null Hypothesis</b>			
-12.331	1.8331	Reject			

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

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\* - The report contents may have been modified or reformatted by end-user of software.

Area: Area 1

X Coord Y C	oord Label	Value	Type	Historical
679530.943	3082575.448	G-36SD	5.3 N	<b>I</b> anual
679606.684	3082692.383	G-37SD	4 N	<b>I</b> anual
679671.317	3082565.925	G-46SD	3.2 N	<b>I</b> anual
679745.982	3082681.386	G-47SD	1.4 N	Ianual
679521.779	3082672.022	J-54SD	1.525	Manual
680108.013	3083101.352	J-57SD	4.8 N	<b>I</b> anual
680413.831	3083297.013	J-58SD	10.2 N	<b>I</b> anual

Area: Area 1

X Coord Y C	Coord Label	Value	Typ	e Historical
679530.943	3082575.448	G-36SD	3.9	Manual
679606.684	3082692.383	G-37SD	2.7	Manual
679671.317	3082565.925	G-46SD	2.7	Manual
679745.982	3082681.386	G-47SD	4.3	Manual
679521.779	3082672.022	J-54SD	2.7	Manual
680108.013	3083101.352	J-57SD	2.7	Manual
680413.831	3083297.013	J-58SD	9.2	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

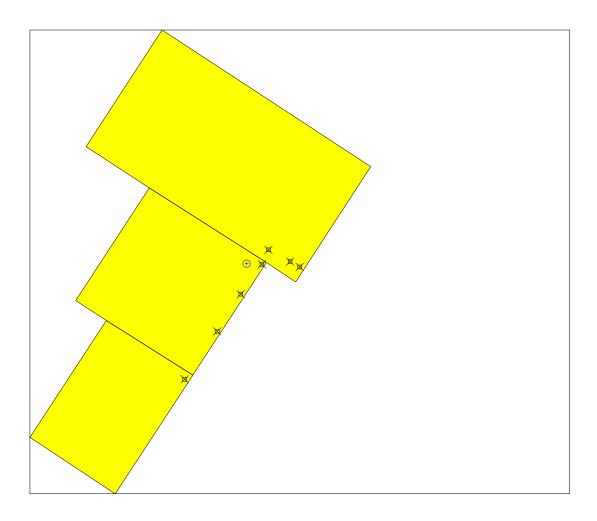
SUMMARY OF	SAMPLING DESIGN
Primary Objective of Design	Compare a site mean to a fixed threshold
Type of Sampling Design	Parametric
Sample Placement (Location) in the Field	Simple random sampling
Working (Null) Hypothesis	The mean value at the site exceeds the threshold
Formula for calculating number of sampling locations	Student's t-test
Calculated total number of samples	2
Number of samples on map <sup>a</sup>	8
Number of selected sample areas b	3
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>
Total cost of sampling <sup>d</sup>	\$2,000.00

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1									
X Coord	Historical								
680077.3540	3083115.5330	J-50S	5.1	Manual	Т				
680141.8730	3083080.8800	J-52S	5.3	Manual	Т				
680170.5600	3083064.6740	J-53S	7.9	Manual	Т				

Area: Area 2								
X Coord	Y Coord	Label	Value	Туре	Historical			
679924.8150	3082872.3490	J-47S	4.4	Manual	Т			
679994.9690	3082983.5100	J-48S	5.5	Manual	Т			
680057.6580	3083072.0750	J-49S	4.5	Manual	Т			
680012.7245	3083074.0231	J-51S	4.6	Random				

Area: Area 3							
X Coord	Y Coord	Label	Value	Туре	Historical		
679827.1150	3082729.7460	J-51S	4.6	Manual	Т		

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null

hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

# **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

n is the number of samples,

S is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analysta	_	Parameter					
Analyte n	S	Δ	α	β	<b>Z</b> <sub>1-α</sub> a	<b>Z<sub>1-β</sub></b> b	
	2	2.1915	283	0.05	0.1	1.64485	1.28155

 $^{\rm a}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

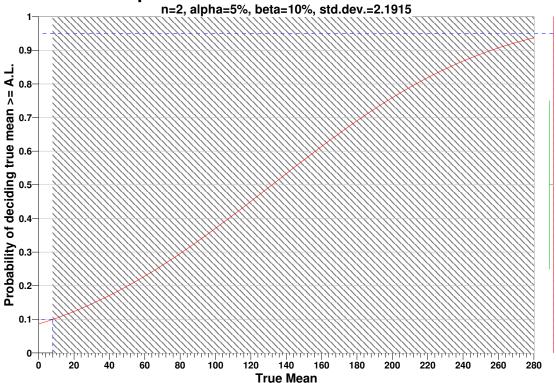
The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at  $1-\alpha$ . If any of the inputs

b This value is automatically calculated by VSP based upon the user defined value of β.

change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



# **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples										
A1 004		α	; <b>=</b> 5	α	=10	α=15				
AL=291		s=4.383	s=2.1915	s=4.383	s=2.1915	s=4.383	s=2.1915			
	β=5	2	2	2	1	1	1			
LBGR=90	β=10	2	2	1	1	1	1			
	β=15	2	2	1	1	1	1			
	β=5	2	2	1	1	1	1			
LBGR=80	β=10	2	2	1	1	1	1			
	β=15	2	2	1	1	1	1			

LBGR=70	β=5	2	2	1	1	1	1
	β=10	2	2	1	1	1	1
	β=15	2	2	1	1	1	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

# **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$2,000.00, which averages out to a per sample cost of \$1,000.00. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION					
Cost Details	Per Analysis	Per Sample	2 Samples		
Field collection costs		\$100.00	\$200.00		
Analytical costs	\$400.00	\$400.00	\$800.00		
Sum of Field & Analytical costs		\$500.00	\$1,000.00		
Fixed planning and validation costs			\$1,000.00		
Total cost			\$2,000.00		

#### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	0	4.4	4.5	4.6	4.6	5.1	5.3	5.5	7.9	

SUMMARY STATIST					TISTI	cs		
n			9					
Min			0					
	M	ах				7.9		
	Rai	nge				7.9		
	Me	an			4	4.6556	6	
Median			4.6					
Variance			4.2028					
StdDev			2.0501					
Std Error				0.68336				
	Skew	ness		-1.2277				
Inte	Interquartile Range				0.95			
Pero				entile	es			
1%	5%	10%	25%	50%	75%	90%	95%	99%
0	0	0	4.45	4.6	5.4	7.9	7.9	7.9

#### **Outlier Test**

Dixon's extreme value test was performed to test whether the lowest value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

DIXON'S OUTLIER TEST				
Dixon Test Statistic	0.8			
Dixon 5% Critical Value	0.554			

The calculated test statistic exceeds the critical value, so the test rejects the null hypothesis that there are no outliers in the data, and concludes that the minimum value 0 is an outlier at the 5% significance level.

SUSPECTED OUTLIERS			
Min	0		

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Dixon's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)				
Shapiro-Wilk Test Statistic	0.7602			
Shapiro-Wilk 5% Critical Value	0.803			

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the minimum value 0, do not appear to follow a normal distribution at the 5% level of significance. Dixon's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

#### **Data Plots**

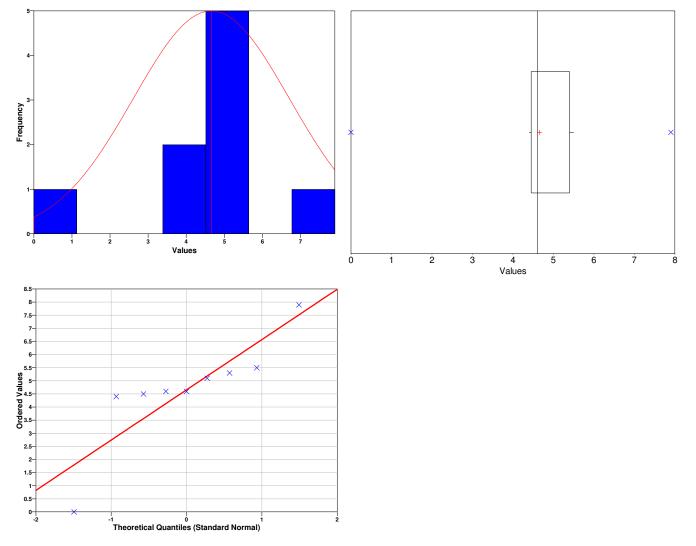
Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5 times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_p$ , for which a

fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/guality/ga-docs.html).

#### lests

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST				
Shapiro-Wilk Test Statistic 0.8085				
Shapiro-Wilk 5% Critical Value	0.829			

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

#### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

**UCLS ON THE MEAN** 

95% Parametric UCL	5.926
95% Non-Parametric (Chebyshev) UCL	7.634

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (7.634) may be a more accurate upper confidence limit on the true mean.

### One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\overline{x} - AL}{SE}$$

# where

x is the sample mean of the n=9 data, AL is the action level or threshold (291),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=8 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST				
t-statistic Critical Value $t_{0.95}$ Null Hypothesis				
-419.03	1.8595	Reject		

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test				
Test Statistic (S+) 95% Critical Value Null Hypothesis				
9	7	Reject		

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\* - The report contents may have been modified or reformatted by end-user of software.

Area: Area 1

X Coord Y C	oord Label	Value	Type	e Historical
679924.815	3082872.349	J-47S	4.4	Manual
679994.969	3082983.51	J-48S	5.5	Manual
680057.658	3083072.075	J-49S	4.5	Manual
680077.354	3083115.533	J-50S	5.1	Manual
679827.115	3082729.746	J-51S	4.6	Manual
680141.873	3083080.88	J-52S	5.3	Manual
680170.56	3083064.674	J-53S	7.9	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

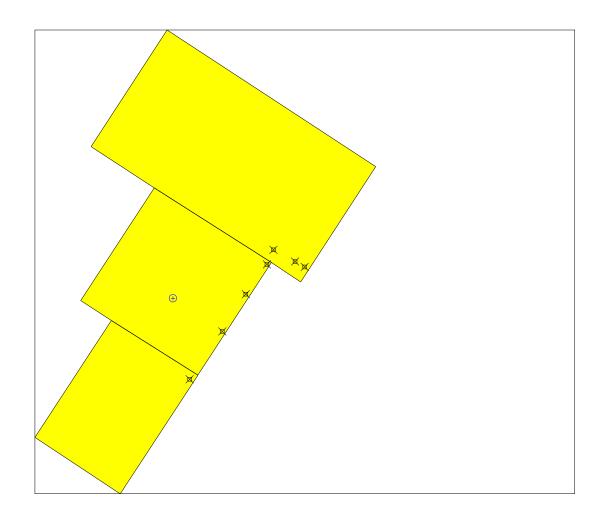
SUMMARY OF SAMPLING DESIGN				
Primary Objective of Design	Compare a site mean to a fixed threshold			
Type of Sampling Design	Parametric			
Sample Placement (Location) in the Field	Simple random sampling			
Working (Null) Hypothesis	The mean value at the site exceeds the threshold			
Formula for calculating number of sampling locations	Student's t-test			
Calculated total number of samples	2			
Number of samples on map <sup>a</sup>	8			
Number of selected sample areas b	3			
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>			
Total cost of sampling <sup>d</sup>	\$2,000.00			

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1								
X Coord	Y Coord	Label	Value	Туре	Historical			
680077.3540	3083115.5330	J-50S	1.4	Manual	Т			
680141.8730	3083080.8800	J-52S	3.6	Manual	Т			
680170.5600	3083064.6740	J-53S	5.9	Manual	Т			

Area: Area 2										
X Coord	Label	Value	Туре	Historical						
679924.8150	3082872.3490	J-47S	4.8	Manual	Т					
679994.9690	3082983.5100	J-48S	5	Manual	Т					
680057.6580	3083072.0750	J-49S	4.1	Manual	Т					
679777.7540	3082971.2341	J-51S	3.5	Random						

Area: Area 3								
X Coord	Y Coord	Label	Value	Туре	Historical			
679827.1150	3082729.7460	J-51S	3.5	Manual	Т			

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null

hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

# **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

is the number of samples,

S is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analyte	_			Parameter				
	"	s	Δ	α	β	<b>Z</b> <sub>1-α</sub> a	<b>Ζ<sub>1-β</sub></b> b	
	2	3	283	0.05	0.1	1.64485	1.28155	

 $^{\rm a}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

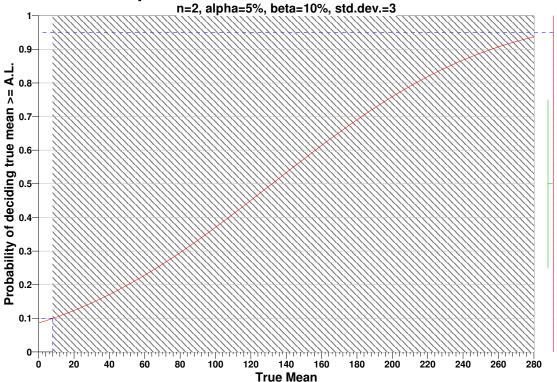
The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at  $1-\alpha$ . If any of the inputs

b This value is automatically calculated by VSP based upon the user defined value of β.

change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



# **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples									
AL=291		α=5		α=10		α=15			
		s=6	s=3	s=6	s=3	s=6	s=3		
LBGR=90	β=5	2	2	2	1	1	1		
	β=10	2	2	2	1	1	1		
	β=15	2	2	2	1	1	1		
LBGR=80	β=5	2	2	1	1	1	1		
	β=10	2	2	1	1	1	1		
	β=15	2	2	1	1	1	1		

	β=5	2	2	1	1	1	1
LBGR=70	β=10	2	2	1	1	1	1
	β=15	2	2	1	1	1	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

### **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$2,000.00, which averages out to a per sample cost of \$1,000.00. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION							
Cost Details	Per Analysis	Per Sample	2 Samples				
Field collection costs		\$100.00	\$200.00				
Analytical costs	\$400.00	\$400.00	\$800.00				
Sum of Field & Analytical costs		\$500.00	\$1,000.00				
Fixed planning and validation costs			\$1,000.00				
Total cost			\$2,000.00				

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	1.4	3.5	3.5	3.6	4.1	4.8	5	5.9		

	SUN	IMARY	/ STA	TISTI	cs			
	n			8				
N	1in				1.4			
N	lax				5.9			
Ra	nge				4.5			
M	ean				3.975	1		
Ме	Median			3.85				
Variance			1.8107					
StdDev			1.3456					
Std Error			0.47575					
Skev	vness		-0.66974					
Interqua	1.45							
		Perc	entile	es				
1% 5%	10%	25%	50%	75%	90%	95%	99%	
1.4 1.4	1.4	3.5	3.85	4.95	5.9	5.9	5.9	

#### **Outlier Test**

Dixon's extreme value test was performed to test whether the lowest value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

DIXON'S OUTLIER TEST				
Dixon Test Statistic	0.58333			
Dixon 5% Critical Value	0.554			

The calculated test statistic exceeds the critical value, so the test rejects the null hypothesis that there are no outliers in the data, and concludes that the minimum value 1.4 is an outlier at the 5% significance level.

SUSPECTED OUTLIERS				
Min	1.4			

Because Dixon's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)					
Shapiro-Wilk Test Statistic	0.8807				
Shapiro-Wilk 5% Critical Value	0.803				

The calculated Shapiro-Wilk test statistic exceeds the 5% Shapiro-Wilk critical value, so the test cannot reject the hypothesis that the data are normal and concludes that the data, excluding the minimum value 1.4, do appear to follow a normal distribution at the 5% level of significance.

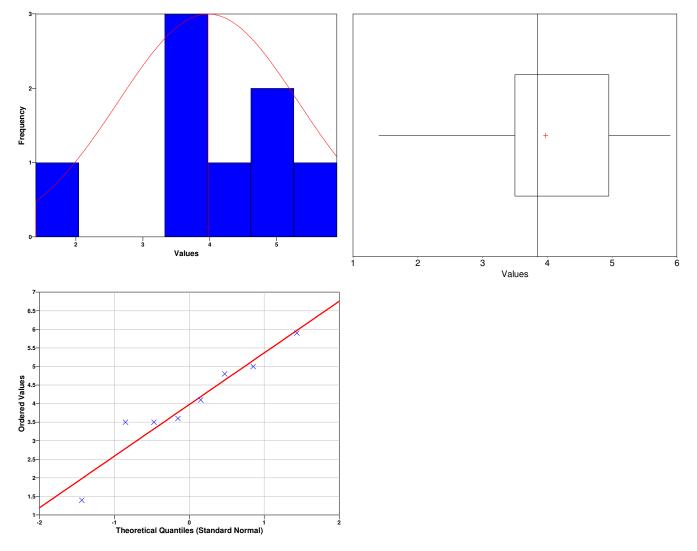
#### **Data Plots**

Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5 times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST					
Shapiro-Wilk Test Statistic 0.9405					
Shapiro-Wilk 5% Critical Value 0.818					

The calculated SW test statistic exceeds the 5% Shapiro-Wilk critical value, so we cannot reject the hypothesis that the data are normal, or in other words the data appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE	MEAN
95% Parametric UCL	4.876

95% Non-Parametric (Chebyshev) UCL 6.049

Because the data appear to be normally distributed according to the goodness-of-fit test performed above, the parametric UCL (4.876) may be a more accurate upper confidence limit on the true mean.

### One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\overline{x} - AL}{SE}$$

where

x is the sample mean of the n=8 data, AL is the action level or threshold (291),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=7 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST					
t-statistic Critical Value t <sub>0.95</sub> Null Hypothesi					
-603.31	1.8946	Reject			

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

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 $^{\star}$  - The report contents may have been modified or reformatted by end-user of software.

Area: Area 1

X Coord Y	Coord Label	Value	Type	e Historical
679924.815	3082872.349	J-47S	4.8	Manual
679994.969	3082983.51	J-48S	5	Manual
680057.658	3083072.075	J-49S	4.1	Manual
680077.354	3083115.533	J-50S	1.4	Manual
679827.115	3082729.746	J-51S	3.5	Manual
680141.873	3083080.88	J-52S	3.6	Manual
680170.56	3083064.674	J-53S	5.9	Manual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

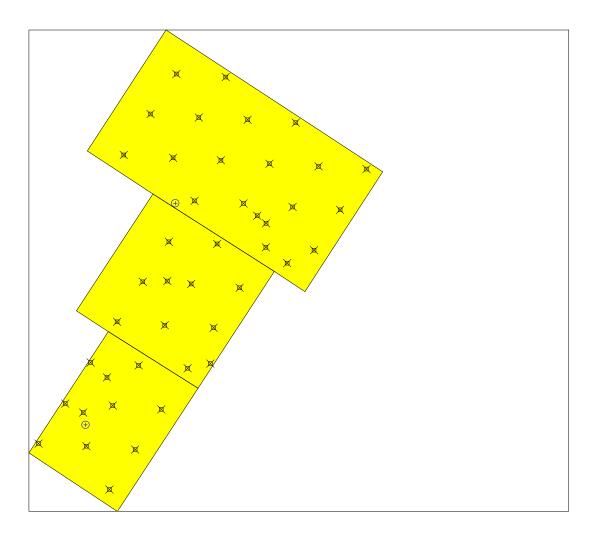
SUMMARY OF	SAMPLING DESIGN
Primary Objective of Design	Compare a site mean to a fixed threshold
Type of Sampling Design	Parametric
Sample Placement (Location) in the Field	Simple random sampling
Working (Null) Hypothesis	The mean value at the site exceeds the threshold
Formula for calculating number of sampling locations	Student's t-test
Calculated total number of samples	21
Number of samples on map <sup>a</sup>	45
Number of selected sample areas b	3
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>
Total cost of sampling <sup>d</sup>	\$11,500.00

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



	Area: Area 1									
X Coord	Y Coord	Label	Value	Туре	Historical					
679638.4660	3083412.5610	G-21SD	134	Manual	Т					
679715.3150	3083530.0190	G-22SD	611	Manual	Т					
679789.8310	3083644.9360	G-23SD	207	Manual	Т					
679780.1110	3083404.0250	G-24SD	53.4	Manual	Т					
679854.2250	3083519.8220	G-25SD	68.2	Manual	Т					
679931.3970	3083636.4060	G-26SD	257.6	Manual	Т					
679841.7740	3083280.2350	G-33SD	91.2	Manual	Т					
679917.7660	3083397.4160	G-34SD	11.2	Manual	Т					
679994.2070	3083513.4470	G-35SD	40	Manual	Т					
679982.2770	3083273.0650	G-42SD	26.7	Manual	Т					
680056.9810	3083387.3300	G-43SD	463	Manual	Т					
680132.6500	3083505.4560	G-44SD	88.2	Manual	Т					
680046.7830	3083146.9990	G-51SD	76.1	Manual	Т					
680123.4320	3083263.0010	G-52SD	95.9	Manual	Т					
680198.0580	3083379.8150	G-53SD	104	Manual	Т					
680185.4120	3083138.8470	G-54SD	128	Manual	Т					

680260.4210	3083254.8070	G-55SD	569	Manual	Т
680335.9700	3083372.0200	G-56SD	86.4	Manual	Т
680022.0080	3083237.4720	J-44SD	29.4	Manual	Т
680047.0950	3083215.9420	J-45SD	91.4	Manual	Т
680108.0130	3083101.3520	J-57SD	802	Manual	Т
679786.3895	3083273.6620	G-30SD	17.2	Random	

	Area: Area 2										
X Coord	Y Coord	Label	Value	Туре	Historical						
679619.1240	3082932.8980	G-30SD	17.2	Manual	Т						
679693.4050	3083047.5490	G-31SD	16.9	Manual	Т						
679768.2260	3083162.7270	G-32SD	52.3	Manual	Т						
679756.1090	3082922.9390	G-39SD	37.8	Manual	Т						
679832.1580	3083041.8710	G-40SD	27.5	Manual	Т						
679906.4210	3083156.7540	G-41SD	37.8	Manual	Т						
679822.0740	3082799.5750	G-48SD	812	Manual	Т						
679896.6120	3082915.6660	G-49SD	24.7	Manual	Т						
679971.2150	3083030.7920	G-50SD	58.3	Manual	Т						
679887.4330	3082812.9360	J-46SD	305	Manual	Т						
679763.3990	3083050.0550	J-56SD	27	Manual	Т						

	Area: Area 3										
X Coord	Y Coord	Label	Value	Туре	Historical						
679393.4380	3082582.9470	G-27SD	29.6	Manual	Т						
679470.2860	3082698.0210	G-28SD	32.25	Manual	Т						
679543.3140	3082816.1060	G-29SD	34.5	Manual	Т						
679530.9430	3082575.4480	G-36SD	119	Manual	Т						
679606.6840	3082692.3830	G-37SD	187	Manual	Т						
679681.4650	3082807.7990	G-38SD	122	Manual	Т						
679597.1880	3082450.9230	G-45SD	96.5	Manual	Т						
679671.3170	3082565.9250	G-46SD	896	Manual	Т						
679745.9820	3082681.3860	G-47SD	64.1	Manual	Т						
679521.7790	3082672.0220	J-54SD	208	Manual	Т						
679590.0920	3082773.1840	J-55SD	59.8	Manual	Т						
679528.3937	3082636.5369		0	Random							

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations.

A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

# **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

*n* is the number of samples,

is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

 $\alpha$  is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\beta}$  is 1- $\beta$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analysta	_				Para	ameter	
Analyte	11	$S \Delta \alpha \beta Z_{1-\alpha} Z_{1-\beta}$					<b>Z</b> <sub>1-β</sub> b
	21	3	2	0.05	0.1	1.64485	1.28155

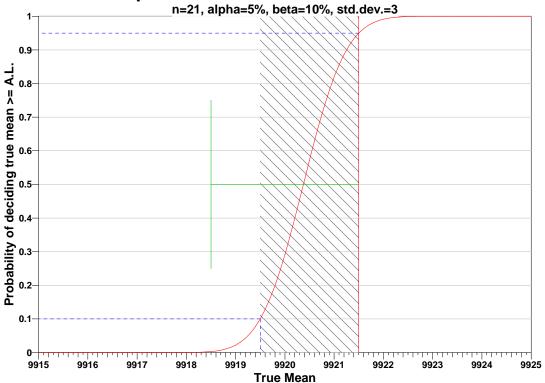
 $^{\text{a}}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ 

<sup>b</sup> This value is automatically calculated by VSP based upon the user defined value of β.

The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at 1- $\alpha$ . If any of the inputs change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



### **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

#### Sensitivity Analysis

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples								
A1 -002	14 E	α	=5	α=	10	α=	15	
AL=992	1.5	s=6	s=3	s=6	s=3	s=6	s=3	
	β=5	2	2	1	1	1	1	
LBGR=90	β=10	2	2	1	1	1	1	
	β=15	2	2	1	1	1	1	
	β=5	2	2	1	1	1	1	
LBGR=80	β=10	2	2	1	1	1	1	
	β=15	2	2	1	1	1	1	
LBGR=70 β=5		2	2	1	1	1	1	

β=10	2	2	1	1	1	1
β=15	2	2	1	1	1	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

## **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$11,500.00, which averages out to a per sample cost of \$547.62. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION									
Cost Details	Per Analysis	Per Sample	21 Samples						
Field collection costs		\$100.00	\$2,100.00						
Analytical costs	\$400.00	\$400.00	\$8,400.00						
Sum of Field & Analytical costs		\$500.00	\$10,500.00						
Fixed planning and validation costs			\$1,000.00						
Total cost			\$11,500.00						

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	0	11.2	16.9	17.2	17.2	24.7	26.7	27	27.5	29.4
10	29.6	29.6	32.25	34.5	37.8	37.8	40	52.3	53.4	58.3
20	59.8	64.1	68.2	76.1	86.4	88.2	91.2	91.4	95.9	96.5
30	104	119	122	128	134	187	207	208	257.6	305
40	463	569	611	802	812	896				

SUMMARY STATISTICS						
n	46					
Min	0					
Max	896					
Range	896					
Mean	159.69					
Median	72.15					
Variance	50869					
StdDev	225.54					
Std Error	33.254					
Skewness	2.195					
Interquartile Range	117.65					

	Percentiles								
1	%	5%	10%	25%	50%	75%	90%	95%	99%
0		13.2	17.2	29.6	72.15	147.3	581.6	808.5	896

#### **Outlier Test**

Rosner's test for multiple outliers was performed to test whether the most extreme value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

In using Rosner's test to detect up to 1 outlier, a test statistic  $R_1$  is calculated, and compared with a critical value  $C_1$  to test the hypothesis that there is one outlier in the data.

ROSNER'S OUTLIER TEST								
k Test Statistic R <sub>k</sub> 5% Critical Value C <sub>k</sub> Significant?								
1	3.231	3.09	Yes					

The test statistic 3.231 exceeded the corresponding critical value, therefore that test is significant and we conclude that the most extreme value is an outlier at the 5% significance level.

SUSPECTE	OUTLIERS
1	896

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Rosner's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)					
Shapiro-Wilk Test Statistic	0.642				
Shapiro-Wilk 5% Critical Value	0.944				

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the most extreme value, do not appear to follow a normal distribution at the 5% level of significance. Rosner's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

### **Data Plots**

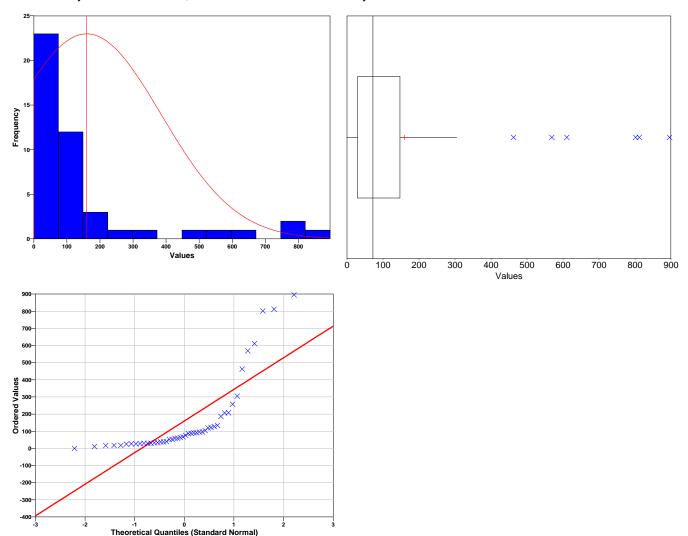
Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5

times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

#### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST				
Shapiro-Wilk Test Statistic	0.6439			
Shapiro-Wilk 5% Critical Value	0.945			

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the

data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE MEAN	
95% Parametric UCL	215.5
95% Non-Parametric (Chebyshev) UCL	304.6

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (304.6) may be a more accurate upper confidence limit on the true mean.

### **One-Sample t-Test**

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\overline{x} - AL}{SE}$$

where

x is the sample mean of the n=46 data,

AL is the action level or threshold (9921.5),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=45 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST					
t-statistic Critical Value t <sub>0.95</sub> Null Hypothe					
-293.55	1.6794	Reject			

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test							
Test Statistic (S+)	95% Critical Value	<b>Null Hypothesis</b>					
46	29	Reject					

This report was automatically produced\* by Visual Sample Plan (VSP) software version 5.000.

Software and documentation available at http://dqo.pnl.gov/vsp

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\* - The report contents may have been modified or reformatted by end-user of software.

Area: Area 1 X Coord Y Coord Label Value Type Historical G-21SD 134 Manual 679638.466 3083412.561 679715.315 3083530.019 G-22SD 611 Manual 679789.831 3083644.936 G-23SD 207 Manual 679780.111 3083404.025 G-24SD 53.4 Manual 679854.225 3083519.822 G-25SD 68.2 Manual 679931.397 3083636.406 G-26SD 257.6 Manual 679393.438 3082582.947 G-27SD 29.6 Manual 679470.286 3082698.021 G-28SD 32.25 Manual 679543.314 3082816.106 G-29SD 34.5 Manual 679619.124 3082932.898 G-30SD 17.2 Manual 3083047.549 G-31SD 16.9 Manual 679693.405 679768.226 3083162.727 G-32SD 52.3 Manual 679841.774 3083280.235 G-33SD 91.2 Manual 679917.766 3083397.416 G-34SD 11.2 Manual 679994.207 3083513.447 G-35SD 40 Manual 3082575.448 G-36SD 119 Manual 679530.943 679606.684 3082692.383 G-37SD 187 Manual 3082807.799 G-38SD 122 Manual 679681.465 3082922.939 G-39SD 37.8 Manual 679756.109 679832.158 3083041.871 G-40SD 27.5 Manual 679906.421 3083156.754 G-41SD 37.8 Manual 679982.277 3083273.065 G-42SD 26.7 Manual G-43SD 463 Manual 680056.981 3083387.33 680132.65 3083505.456 G-44SD 88.2 Manual 679597.188 3082450.923 G-45SD 96.5 Manual 3082565.925 G-46SD 896 Manual 679671.317 3082681.386 G-47SD 64.1 Manual 679745.982 3082799.575 G-48SD 812 Manual 679822.074 3082915.666 G-49SD 24.7 Manual 679896.612 679971.215 3083030.792 G-50SD 58.3 Manual 680046.783 3083146.999 G-51SD 76.1 Manual 680123.432 3083263.001 G-52SD 95.9 Manual 680198.058 3083379.815 G-53SD 104 Manual 3083138.847 G-54SD 128 Manual 680185.412 680260.421 3083254.807 G-55SD 569 Manual 680335.97 3083372.02 G-56SD 86.4 Manual J-44SD 29.4 Manual 680022.008 3083237.472 3083215.942 J-45SD 680047.095 91.4 Manual 679887.433 3082812.936 J-46SD 305 Manual 3082672.022 J-54SD 679521.779 208 Manual 679590.092 3082773.184 J-55SD 59.8 Manual 3083050.055 J-56SD 27 679763.399 Manual 680108.013 3083101.352 J-57SD 802 Manual

3083297.013 J-58SD

96

Manual

680413.831

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

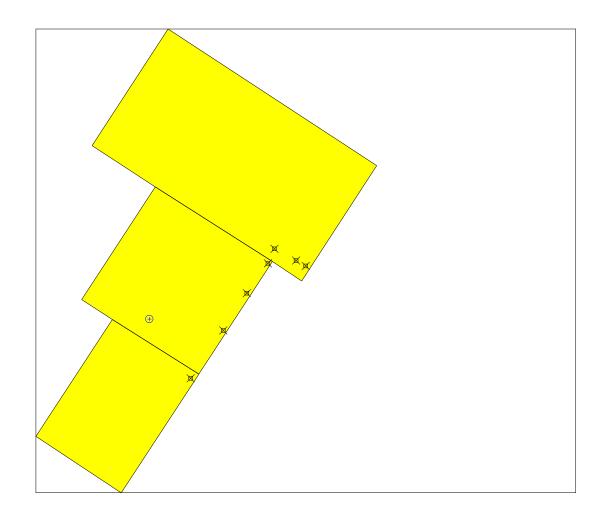
SUMMARY OF SAMPLING DESIGN						
Primary Objective of Design	Compare a site mean to a fixed threshold					
Type of Sampling Design	Parametric					
Sample Placement (Location) in the Field	Simple random sampling					
Working (Null) Hypothesis	The mean value at the site exceeds the threshold					
Formula for calculating number of sampling locations	Student's t-test					
Calculated total number of samples	2					
Number of samples on map <sup>a</sup>	8					
Number of selected sample areas b	3					
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>					
Total cost of sampling <sup>d</sup>	\$2,000.00					

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1								
X Coord Y Coord Label Value Type Historic								
680077.3540	3083115.5330	J-50S	13.7	Manual	Т			
680141.8730	3083080.8800	J-52S	8.9	Manual	Т			
680170.5600	3083064.6740	J-53S	17.5	Manual	Т			

Area: Area 2								
X Coord	Historical							
679924.8150	3082872.3490	J-47S	35.8	Manual	Т			
679994.9690	3082983.5100	J-48S	10.2	Manual	Т			
680057.6580	3083072.0750	J-49S	18.2	Manual	Т			
679704.1121	3082906.1307	J-51S	11.9	Random				

Area: Area 3						
X Coord Y Coord Label Value Type Historic						
679827.1150	3082729.7460	J-51S	11.9	Manual	Т	

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null

hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

### **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

is the number of samples,

S is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analysta	_			Paraı	nete	r	
Analyte	n	S	Δ	α	β	<b>Z</b> <sub>1-α</sub> <sup>a</sup>	<b>Z<sub>1-β</sub></b> b
	2	8.6362	9913.5	0.05	0.1	1.64485	1.28155

 $^{\rm a}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

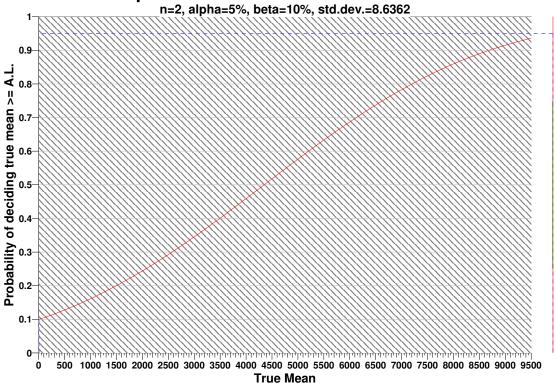
The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at  $1-\alpha$ . If any of the inputs

b This value is automatically calculated by VSP based upon the user defined value of β.

change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



### **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- 4. the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

Number of Samples									
AL=9921.5		α=5		α=	10	α=15			
		s=17.2724	s=8.6362	s=17.2724	s=8.6362	s=17.2724	s=8.6362		
	β=5	2	2	1	1	1	1		
LBGR=90	β=10	2	2	1	1	1	1		
	β=15	2	2	1	1	1	1		
	β=5	2	2	1	1	1	1		
LBGR=80	β=10	2	2	1	1	1	1		
	β=15	2	2	1	1	1	1		

	β=5	2	2	1	1	1	1
LBGR=70	β=10	2	2	1	1	1	1
	β=15	2	2	1	1	1	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

### **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$2,000.00, which averages out to a per sample cost of \$1,000.00. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION					
Cost Details	Per Analysis	Per Sample	2 Samples		
Field collection costs		\$100.00	\$200.00		
Analytical costs	\$400.00	\$400.00	\$800.00		
Sum of Field & Analytical costs		\$500.00	\$1,000.00		
Fixed planning and validation costs			\$1,000.00		
Total cost			\$2,000.00		

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	8.9	10.2	11.9	11.9	13.7	17.5	18.2	35.8		

		SUM	MMAR'	Y STA	TISTIC	S		
n				8				
	M	lin				8.9		
	М	ах				35.8		
	Ra	nge				26.9		
	Ме	ean			1	6.012		
Median			12.8					
Variance			74.584					
StdDev				8.6362				
Std Error				3.0534				
	Skev	vness		2.0859				
Inte	erquar	tile Ra	nge	7.4				
			Per	centil	es			
1%	5%	10%	25%	50%	75%	90%	95%	99%
8.9	8.9	8.9	10.63	12.8	18.02	35.8	35.8	35.8

#### **Outlier Test**

Dixon's extreme value test was performed to test whether the lowest value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

DIXON'S OUTLIER TEST			
Dixon Test Statistic	0.13978		
Dixon 5% Critical Value	0.554		

The calculated test statistic does not exceed the critical value, so the test cannot reject the null hypothesis that there are no outliers in the data, and concludes that the minimum value 8.9 is not an outlier at the 5% significance level.

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Dixon's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)					
Shapiro-Wilk Test Statistic	0.7492				
Shapiro-Wilk 5% Critical Value	0.803				

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the minimum value 8.9, do not appear to follow a normal distribution at the 5% level of significance. Dixon's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

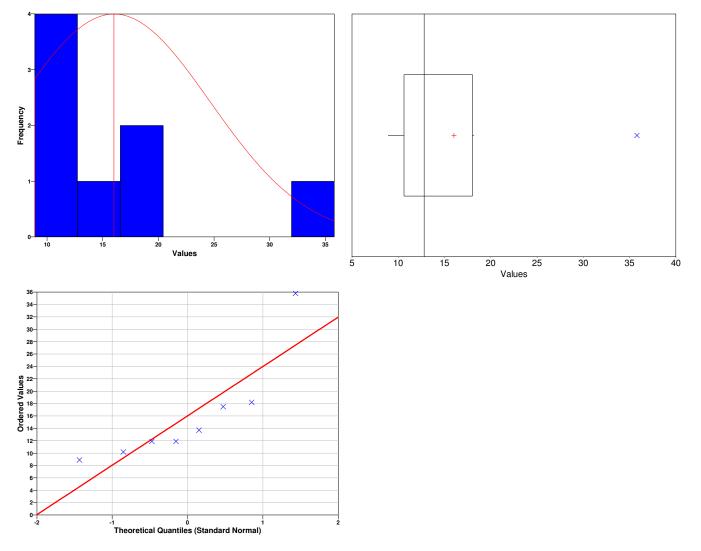
### **Data Plots**

Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5 times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/ga-docs.html).

### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST				
Shapiro-Wilk Test Statistic	0.7576			
Shapiro-Wilk 5% Critical Value 0.818				

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE	MEAN
95% Parametric UCL	21.8

# 95% Non-Parametric (Chebyshev) UCL 29.32

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (29.32) may be a more accurate upper confidence limit on the true mean.

### One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\overline{x} - AL}{SE}$$

where

x is the sample mean of the n=8 data, AL is the action level or threshold (9921.5),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=7 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST				
t-statistic Critical Value t <sub>0.95</sub> Null Hypothesis				
-3244.1	1.8946	Reject		

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test				
Test Statistic (S+) 95% Critical Value Null Hypothesis				
8	6	Reject		

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Area: Area 1

X Coord Y C	Coord Label	Value	Type	Historical
679924.815	3082872.349	J-47S	35.8 Ma	ınual
679994.969	3082983.51	J-48S	10.2 Ma	ınual
680057.658	3083072.075	J-49S	18.2 Ma	ınual
680077.354	3083115.533	J-50S	13.7 Ma	ınual
679827.115	3082729.746	J-51S	11.9 Ma	ınual
680141.873	3083080.88	J-52S	8.9 Ma	ınual
680170.56	3083064.674	J-53S	17.5 Ma	ınual

### Random sampling locations for comparing a mean with a fixed threshold (parametric)

#### **Summary**

This report summarizes the sampling design used, associated statistical assumptions, as well as general guidelines for conducting post-sampling data analysis. Sampling plan components presented here include how many sampling locations to choose and where within the sampling area to collect those samples. The type of medium to sample (i.e., soil, groundwater, etc.) and how to analyze the samples (in-situ, fixed laboratory, etc.) are addressed in other sections of the sampling plan.

The following table summarizes the sampling design developed. A figure that shows sampling locations in the field and a table that lists sampling location coordinates are also provided below.

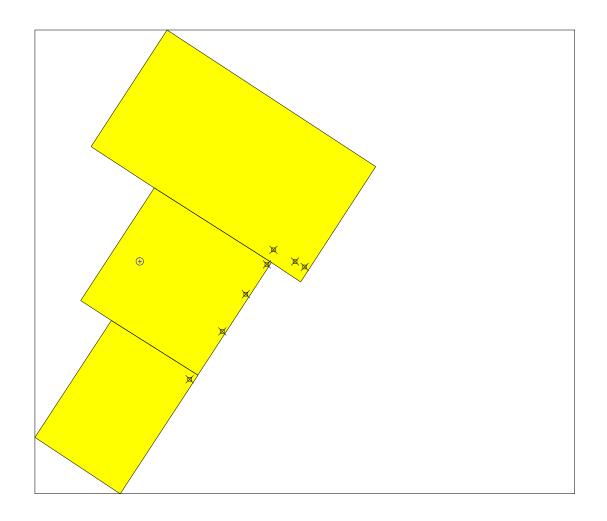
SUMMARY OF	SAMPLING DESIGN
Primary Objective of Design	Compare a site mean to a fixed threshold
Type of Sampling Design	Parametric
Sample Placement (Location) in the Field	Simple random sampling
Working (Null) Hypothesis	The mean value at the site exceeds the threshold
Formula for calculating number of sampling locations	Student's t-test
Calculated total number of samples	2
Number of samples on map <sup>a</sup>	8
Number of selected sample areas b	3
Specified sampling area <sup>c</sup>	602131.68 m <sup>2</sup>
Total cost of sampling <sup>d</sup>	\$2,000.00

<sup>&</sup>lt;sup>a</sup> This number may differ from the calculated number because of 1) grid edge effects, 2) adding judgment samples, or 3) selecting or unselecting sample areas.

<sup>&</sup>lt;sup>b</sup> The number of selected sample areas is the number of colored areas on the map of the site. These sample areas contain the locations where samples are collected.

<sup>&</sup>lt;sup>c</sup> The sampling area is the total surface area of the selected colored sample areas on the map of the site.

<sup>&</sup>lt;sup>d</sup> Including measurement analyses and fixed overhead costs. See the Cost of Sampling section for an explanation of the costs presented here.



Area: Area 1										
X Coord	Y Coord	Label	Value	Туре	Historical					
680077.3540	3083115.5330	J-50S	23.9	Manual	Т					
680141.8730	3083080.8800	J-52S	32.8	Manual	Т					
680170.5600	3083064.6740	J-53S	279	Manual	Т					

Area: Area 2										
X Coord	Y Coord	Label	Value	Туре	Historical					
679924.8150	3082872.3490	J-47S	346	Manual	Т					
679994.9690	3082983.5100	J-48S	44	Manual	Т					
680057.6580	3083072.0750	J-49S	32.9	Manual	Т					
679678.8319	3083081.0582	J-51S	66.6	Random						

Area: Area 3									
X Coord	Y Coord	Label	Value	Туре	Historical				
679827.1150	3082729.7460	J-51S	66.6	Manual	Т				

# **Primary Sampling Objective**

The primary purpose of sampling at this site is to compare a mean value with a fixed threshold. The working hypothesis (or 'null' hypothesis) is that the mean value at the site is equal to or exceeds the threshold. The alternative hypothesis is that the mean value is less than the threshold. VSP calculates the number of samples required to reject the null

hypothesis in favor of the alternative hypothesis, given a selected sampling approach and inputs to the associated equation.

# **Selected Sampling Approach**

A parametric random sampling approach was used to determine the number of samples and to specify sampling locations. A parametric formula was chosen because the conceptual model and historical information (e.g., historical data from this site or a very similar site) indicate that parametric assumptions are reasonable. These assumptions will be examined in post-sampling data analysis.

Both parametric and non-parametric approaches rely on assumptions about the population. However, non-parametric approaches typically require fewer assumptions and allow for more uncertainty about the statistical distribution of values at the site. The trade-off is that if the parametric assumptions are valid, the required number of samples is usually less than the number of samples required by non-parametric approaches.

Locating the sample points randomly provides data that are separated by many distances, whereas systematic samples are all equidistant apart. Therefore, random sampling provides more information about the spatial structure of the potential contamination than systematic sampling does. As with systematic sampling, random sampling also provides information regarding the mean value, but there is the possibility that areas of the site will not be represented with the same frequency as if uniform grid sampling were performed.

### **Number of Total Samples: Calculation Equation and Inputs**

The equation used to calculate the number of samples is based on a Student's t-test. For this site, the null hypothesis is rejected in favor of the alternative hypothesis if the sample mean is sufficiently smaller than the threshold. The number of samples to collect is calculated so that 1) there will be a high probability  $(1-\beta)$  of rejecting the null hypothesis if the alternative hypothesis is true and 2) a low probability  $(\alpha)$  of rejecting the null hypothesis is true.

The formula used to calculate the number of samples is:

$$n = \frac{S^2}{\Delta^2} \left( Z_{1-\alpha} + Z_{1-\beta} \right)^2 + 0.5 Z_{1-\alpha}^2$$

where

is the number of samples,

S is the estimated standard deviation of the measured values including analytical error,

 $\Delta$  is the width of the gray region,

is the acceptable probability of incorrectly concluding the site mean is less than the threshold,

is the acceptable probability of incorrectly concluding the site mean exceeds the threshold,

is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ , is the value of the standard normal distribution such that the proportion of the distribution less than  $Z_{1-\alpha}$  is 1- $\alpha$ .

The values of these inputs that result in the calculated number of sampling locations are:

Analyta	_		Parameter						
Analyte	11	S	Δ	α	β	$Z_{1-\alpha}^{a}$	<b>Z<sub>1-β</sub></b> b		
	2	126.31	9913.5	0.05	0.1	1.64485	1.28155		

 $^{\rm a}$  This value is automatically calculated by VSP based upon the user defined value of  $\alpha$ .

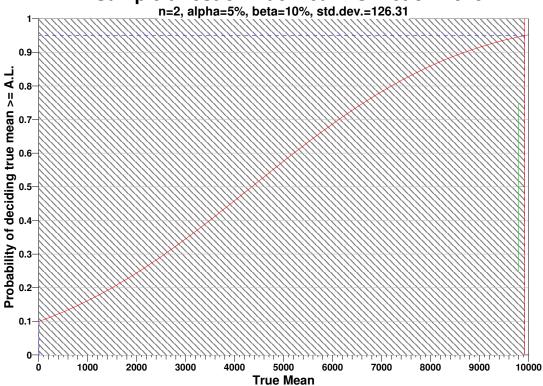
The following figure is a performance goal diagram, described in EPA's QA/G-4 guidance (EPA, 2000). It shows the probability of concluding the sample area is dirty on the vertical axis versus a range of possible true mean values for the site on the horizontal axis. This graph contains all of the inputs to the number of samples equation and pictorially represents the calculation.

The red vertical line is shown at the threshold (action limit) on the horizontal axis. The width of the gray shaded area is equal to  $\Delta$ ; the upper horizontal dashed blue line is positioned at 1- $\alpha$  on the vertical axis; the lower horizontal dashed blue line is positioned at  $\beta$  on the vertical axis. The vertical green line is positioned at one standard deviation below the threshold. The shape of the red curve corresponds to the estimates of variability. The calculated number of samples results in the curve that passes through the lower bound of  $\Delta$  at  $\beta$  and the upper bound of  $\Delta$  at  $1-\alpha$ . If any of the inputs

b This value is automatically calculated by VSP based upon the user defined value of β.

change, the number of samples that result in the correct curve changes.

# 1-Sample t-Test of True Mean vs. Action Level



### **Statistical Assumptions**

The assumptions associated with the formulas for computing the number of samples are:

- 1. the sample mean is normally distributed (this happens if the data are roughly symmetric and the sample size is 30 or more; for skewed data sets, additional samples are required for the sample mean to be normally distributed),
- 2. the variance estimate,  $S^2$ , is reasonable and representative of the population being sampled,
- 3. the population values are not spatially or temporally correlated, and
- the sampling locations will be selected randomly.

The first three assumptions will be assessed in a post data collection analysis. The last assumption is valid because the sample locations were selected using a random process.

# **Sensitivity Analysis**

The sensitivity of the calculation of number of samples was explored by varying the standard deviation, lower bound of gray region (% of action level), beta (%), probability of mistakenly concluding that  $\mu$  > action level and alpha (%), probability of mistakenly concluding that  $\mu$  < action level and examining the resulting changes in the number of samples. The following table shows the results of this analysis.

	Number of Samples											
AL=9921.5		α=	=5	α=	:10	α=15						
		s=252.62	s=126.31	s=252.62	s=126.31	s=252.62	s=126.31					
	β=5	3	2	2	1	2	1					
LBGR=90	β=10	2	2	2	1	1	1					
	β=15	2	2	2	1	1	1					
	β=5	2	2	1	1	1	1					
LBGR=80	β=10	2	2	1	1	1	1					
	β=15	2	2	1	1	1	1					

	β=5	2	2	1	1	1	1
LBGR=70	β=10	2	2	1	1	1	1
	β=15	2	2	1	1	1	1

s = Standard Deviation

LBGR = Lower Bound of Gray Region (% of Action Level)

 $\beta$  = Beta (%), Probability of mistakenly concluding that  $\mu$  > action level

 $\alpha$  = Alpha (%), Probability of mistakenly concluding that  $\mu$  < action level

AL = Action Level (Threshold)

### **Cost of Sampling**

The total cost of the completed sampling program depends on several cost inputs, some of which are fixed, and others that are based on the number of samples collected and measured. Based on the numbers of samples determined above, the estimated total cost of sampling and analysis at this site is \$2,000.00, which averages out to a per sample cost of \$1,000.00. The following table summarizes the inputs and resulting cost estimates.

COST INFORMATION										
Cost Details	Per Analysis	Per Sample	2 Samples							
Field collection costs		\$100.00	\$200.00							
Analytical costs	\$400.00	\$400.00	\$800.00							
Sum of Field & Analytical costs		\$500.00	\$1,000.00							
Fixed planning and validation costs			\$1,000.00							
Total cost			\$2,000.00							

### **Data Analysis**

The following data points were entered by the user for analysis.

Rank	1	2	3	4	5	6	7	8	9	10
0	23.9	32.8	32.9	44	66.6	66.6	279	346		

		SUI	MMAR'	Y STA	TISTIC	S		
n						8		
Min						23.9		
	М	ах				346		
	Ra	nge			;	322.1		
	Ме	ean			1	11.47	i	
	Ме	dian				55.3		
	Vari	ance		15954				
	Std	Dev		126.31				
	Std	Error		44.657				
	Skev	vness		1.4621				
Inte	erquar	tile Ra	nge	193.08				
			Per	centil	es			
1%	5%	10%	25%	50%	75%	90%	95%	99%
23.9	23.9	23.9	32.82	55.3	225.9	346	346	346

#### **Outlier Test**

Dixon's extreme value test was performed to test whether the lowest value is a statistical outlier. The test was conducted at the 5% significance level.

Data should not be excluded from analysis solely on the basis of the results of this or any other statistical test. If any values are flagged as possible outliers, further investigation is recommended to determine whether there is a plausible explanation that justifies removing or replacing them.

DIXON'S OUTLIER TEST					
Dixon Test Statistic	0.034888				
Dixon 5% Critical Value	0.554				

The calculated test statistic does not exceed the critical value, so the test cannot reject the null hypothesis that there are no outliers in the data, and concludes that the minimum value 23.9 is not an outlier at the 5% significance level.

A normal distribution test indicated that the data do not appear to be normally distributed, so further investigation is recommended before using the results of this test. Because Dixon's test can be used only when the data without the suspected outlier are approximately normally distributed, a Shapiro-Wilk test for normality was performed at a 5% significance level.

NORMAL DISTRIBUTION TEST (excluding outliers)							
Shapiro-Wilk Test Statistic	0.7241						
Shapiro-Wilk 5% Critical Value	0.803						

The calculated Shapiro-Wilk test statistic is less than the 5% Shapiro-Wilk critical value, so the test rejects the hypothesis that the data are normal and concludes that the data, excluding the minimum value 23.9, do not appear to follow a normal distribution at the 5% level of significance. Dixon's test may not be appropriate if the assumption of normally distributed data is not justified for this data set. Examine the Q-Q plot displayed below to further assess the normality of the data.

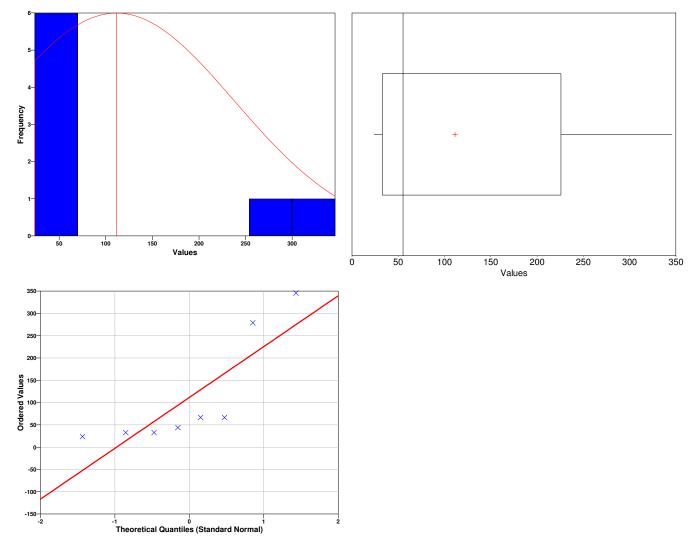
### **Data Plots**

Graphical displays of the data are shown below.

The Histogram is a plot of the fraction of the n observed data that fall within specified data "bins." A histogram is generated by dividing the x axis (range of the observed data values) into "bins" and displaying the number of data in each bin as the height of a bar for the bin. The area of the bar is the fraction of the n data values that lie within the bin. The sum of the fractions for all bins equals one. A histogram is used to assess how the n data are distributed (spread) over their range of values. If the histogram is more or less symmetric and bell shaped, then the data may be normally distributed.

The Box and Whiskers plot is composed of a central box divided by a line, and with two lines extending out from the box, called the "whiskers". The line through the box is drawn at the median of the n data observed. The two ends of the box represent the 25<sup>th</sup> and 75<sup>th</sup> percentiles of the n data values, which are also called the lower and upper quartiles, respectively, of the data set. The sample mean (mean of the n data) is shown as a "+" sign. The upper whisker extends to the largest data value that is less than the upper quartile plus 1.5 times the interquartile range (upper quartile minus the lower quartile). The lower whisker extends to the smallest data value that is greater than the lower quartile minus 1.5 times the interquartile range. Extreme data values (greater or smaller than the ends of the whiskers) are plotted individually as blue Xs. A Box and Whiskers plot is used to assess the symmetry of the distribution of the data set. If the distribution is symmetrical, the box is divided into two equal halves by the median, the whiskers will be the same length, and the number of extreme data points will be distributed equally on either end of the plot.

The Q-Q plot graphs the quantiles of a set of n data against the quantiles of a specific distribution. We show here only the Q-Q plot for an assumed normal distribution. The  $p^{th}$  quantile of a distribution of data is the data value,  $x_n$ , for which a fraction p of the distribution is less than  $x_n$ . If the data plotted on the normal distribution Q-Q plot closely follow a straight line, even at the ends of the line, then the data may be assumed to be normally distributed. If the data points deviate substantially from a linear line, then the data are not normally distributed.



For more information on these plots consult Guidance for Data Quality Assessment, EPA QA/G-9, pgs 2.3-1 through 2.3-12. (http://www.epa.gov/quality/qa-docs.html).

### **Tests**

A goodness-of-fit test was performed to test whether the data set had been drawn from an underlying normal distribution. The Shapiro-Wilk (SW) test was used to test the null hypothesis that the data are normally distributed. The test was conducted at the 5% significance level, i.e., the probability the test incorrectly rejects the null hypothesis was set at 0.05.

NORMAL DISTRIBUTION TEST						
Shapiro-Wilk Test Statistic 0.7019						
Shapiro-Wilk 5% Critical Value	0.818					

The calculated SW test statistic is less than the 5% Shapiro-Wilk critical value, so we can reject the hypothesis that the data are normal, or in other words the data do not appear to follow a normal distribution at the 5% level of significance. The Q-Q plot displayed above should be used to further assess the normality of the data.

### **Upper Confidence Limit on the True Mean**

Two methods were used to compute the upper confidence limit (UCL) on the mean. The first is a parametric method that assumes a normal distribution. The second is the Chebyshev method, which requires no distributional assumption.

UCLs ON THE MEAN		
95% Parametric UCL	196.1	

95% Non-Parametric (Chebyshev) UCL 306.1

Because the data do not appear to be normally distributed according to the goodness-of-fit test performed above, the non-parametric UCL (306.1) may be a more accurate upper confidence limit on the true mean.

### One-Sample t-Test

A one-sample t-test was performed to compare the sample mean to the action level. The null hypothesis used is that the true mean equals or exceeds the action level (AL). The t-test was conducted at the 5% significance level. The sample value *t* was computed using the following equation:

$$t = \frac{\bar{x} - AL}{SE}$$

where

x is the sample mean of the n=8 data, AL is the action level or threshold (9921.5),

SE is the standard error = (standard deviation) / (square root of n).

This t was then compared with the critical value  $t_{0.95}$ , where  $t_{0.95}$  is the value of the t distribution with n-1=7 degrees of freedom for which the proportion of the distribution to the left of  $t_{0.95}$  is 0.95. The null hypothesis will be rejected if  $t < -t_{0.95}$ .

ONE-SAMPLE t-TEST				
t-statistic	Critical Value t <sub>0.95</sub>	Null Hypothesis		
-219.67	1.8946	Reject		

The test rejected the null hypothesis that the mean value at the site exceeds the threshold, therefore conclude the true mean is less than the threshold.

Because the data do not appear to be normally distributed, the MARSSIM Sign Test might be preferred over the One Sample t-Test. The following table represents the results of the MARSSIM Sign Test using the current data:

MARSSIM Sign Test					
Test Statistic (S+)	95% Critical Value	<b>Null Hypothesis</b>			
8	6	Reject			

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 $^{\star}$  - The report contents may have been modified or reformatted by end-user of software.

Area: Area 1

X Coord Y C	Coord Label	Value	Type Historical
679924.815	3082872.349	J-47S	346 Manual
679994.969	3082983.51	J-48S	44 Manual
680057.658	3083072.075	J-49S	32.9 Manual
680077.354	3083115.533	J-50S	23.9 Manual
679827.115	3082729.746	J-51S	66.6 Manual
680141.873	3083080.88	J-52S	32.8 Manual
680170.56	3083064.674	J-53S	279 Manual